Glacier hydrology notes

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Contents

1 Glacier hydrology overview and context

- Glaciers and ice sheets are made of frozen water (mostly pure water here on Earth, but many other kinds of ice on different planets and icy satellites).
- Liquid water flows on, through, and under glaciers and ice sheets. Glacier water flow can broadly be classified as supraglacial (on top), englacial (inside), and subglacial (below the ice) hydrology. These pieces are all connected.
- Water influences the behavior of the glacier by lubricating the bed (affecting sliding velocity, which depends on effective pressure). But the relation between meltwater and sliding velocity is not as straightforward as you

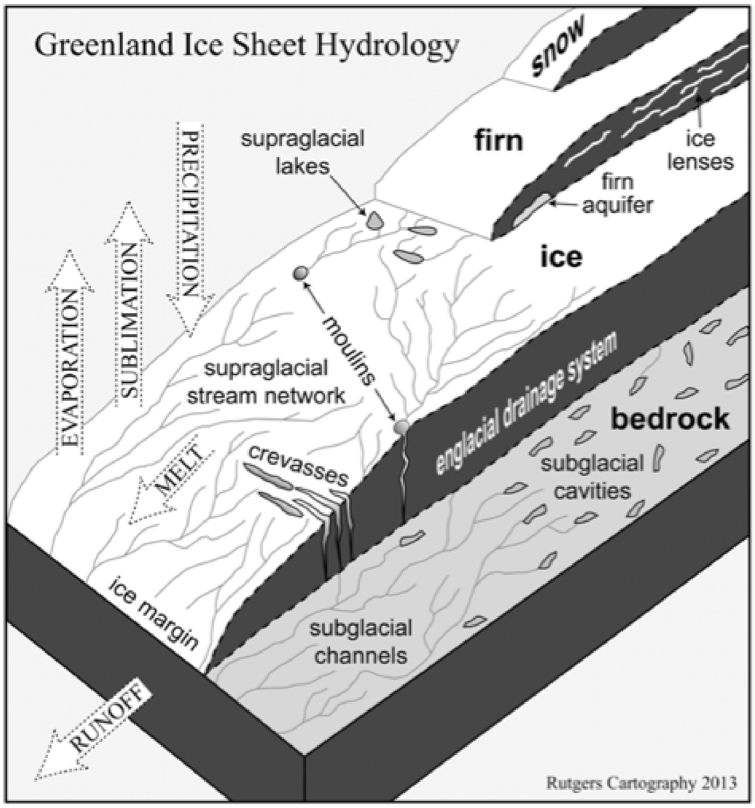

Figure 1: Schematic of Greenland Ice Sheet hydrology (Rennermalm et al., 2013).

might think! More melt can lead to increased pressure at the bed, but with enough meltwater and the right conditions, the ice melts to form more efficient drainage channels, actually leading to a decrease in water pressure at the bed and a deceleration of the ice. This is complicated and depends on topography, ice thickness, and the rate, distribution, and magnitude of meltwater reaching the bed, among other things.

- When lakes dammed by the glacier (like Hidden Creek Lake here at Kennicott Glacier; Bartholomaus et al., 2008, Bartholomaus et al., 2011, Armstrong and Anderson, 2020) or proglacial lakes break through their dams, outburst floods occur - which can be fun for kayaking, but dangerous for infrastructure and human settlements. Catastrophic examples in the Himalayas.

2 Supraglacial hydrology

- Water on the surface of a glacier (from Latin, the prefix supra = above).
- Supraglacial lakes: Vary in size from small ponds to massive lakes. May eventually form a moulin and drain to the bed via hydrofracture (Das et al., 2008).
- Supraglacial rivers: Vary in size from small streams that you can step over, to large uncrossable rivers (Smith et al., 2015).
- Moulins: Vertical shafts that drain water from the surface into the ice. The actual shape is complex (Covington et al., 2020).
- Crevasses: Cracks that form where the glacier is in tension (getting "pulled apart"). Crevasses can form from the surface or from the bed. (Colgan et al., 2016)
- Cryoconite holes: Water-filled holes on the surface formed by melting around soil or rocks on the ice surface. Home to diverse microbial populations (P. Sommers et al., 2019).
- Saturated crust: Porous shallow water flow across the surface, connecting cryoconite holes (Cooper et al., 2018).
- Some (most or all in some places) surface water enters the ice sheet or glacier and eventually drains to the bed (Yang and Smith, 2016). In some places, water percolates into the firn and refreezes, forming refrozen lenses or layers, or is maintained as liquid water in a firn aquifer (Forster et al., 2014).

3 Englacial hydrology

- Liquid water that flows through a glacier (en- meaning "in" or "within").
- Moulins: vertical shafts from ice surface to bed, large range in size, shape can be complex and largely unknown (Covington et al., 2020).
- Conduits: "pipes" within the ice.
- Firn aquifers: Water percolates into the firn but does not refreeze - remains as a perched liquid aquifer, may eventually drain via crevasses to the bed. These exist in mountain glaciers and in several places in Greenland (first one in Greenland discovered in the southeast upstream of Helheim Glacier, Forster et al., 2014,Miège et al., 2016).

4 Subglacial hydrology

- Water that flows beneath a glacier (sub- prefix meaning "under" or "below").

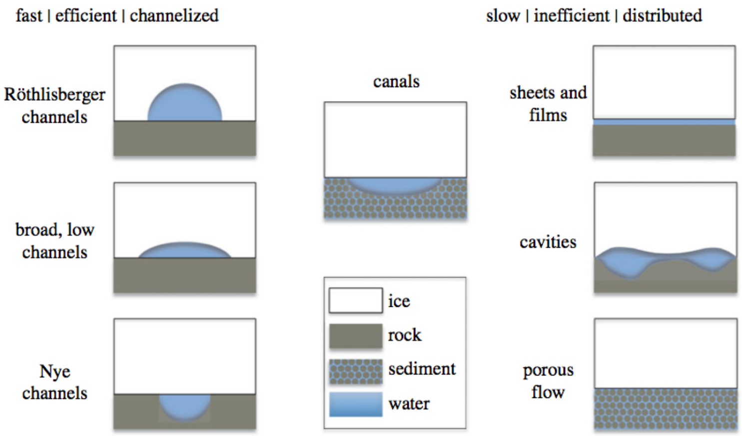

Figure 2: Different subglacial drainage types (Flowers, 2015).

4.1 Various subglacial drainage features

- Channels (Röthlisberger, 1972, **nye**)
- Canals (Fowler and Walder, 1993, Walder and Fowler, 1994)
- Sheets, films (Weertman, 1957)
- Linked cavities (Kamb, 1987)
- Porous till, hyporheic zone, deeper groundwater (Blankenship et al., 1986, Shoemaker, 1986, Gustafson et al., 2022)
- Weakly connected or disconnected regions (Hoffman et al., 2016, Rada and Schoof, 2018, Mejıa et al., 2021)
- What else should be considered?

4.2 Mechanisms of drainage system opening and closing

- Opening by melt at the bed (due to geothermal heat, frictional heat from sliding, and kinetic energy of the water flow dissipated into thermal energy)
- Opening by sliding over bumps in the bed (creating cavities in the lee of the bumps)
- Closing by creep of the ice

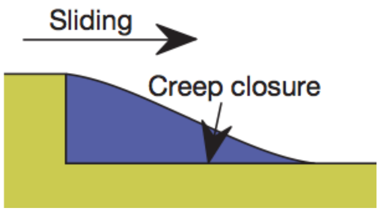

Figure 3: Opening of subglacial cavities by sliding over bed bumps and closing due to creep (Schoof, 2010).

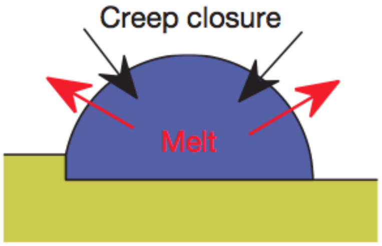

Figure 4: Opening by melt and closing of subglacial drainage system by creep closure of the ice (Schoof, 2010).

4.3 Concepts of effective pressure and hydraulic head

- Sliding velocity depends on effective pressure N (the difference between ice overburden pressure p_i and subglacial water pressure p_w):

$$N = p_i - p_w \tag{1}$$

$$p_i = \rho_i g H \tag{2}$$

$$p_w = \rho_w g(h - z_b) \tag{3}$$

  where ρ_i and ρ_w are density of ice and water, respectively, g is gravitational acceleration, H is ice thickness, h is hydraulic head, and z_b is bed elevation.

- Lower effective pressure = higher water pressure → faster sliding.
- Higher effective pressure = lower water pressure → slower sliding.
- Subglacial water flows according to hydraulic potential or head gradient (dh/dx). This means that water can flow uphill under a glacier.
- Hydraulic head h is defined as the pressure head plus the elevation head:

$$h = \frac{p_w}{\rho_w g} + z_b \tag{4}$$

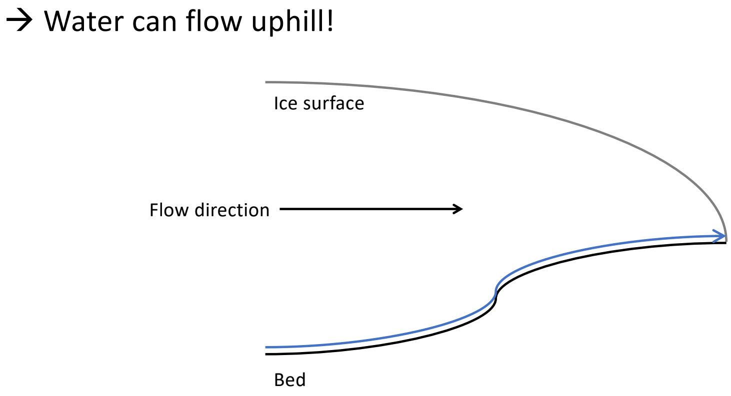

Figure 5: Illustration of subglacial water flowing up hill.

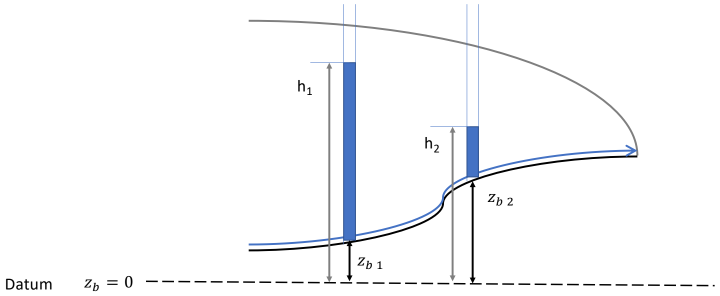

Figure 6: Illustration of hydraulic head as measured relative to an arbitrary datum.

- Hydraulic head is closely related to hydraulic potential ϕ (which is also frequently used in models):

$$\phi = p_w + \rho_w g z_b = \rho_w g(h - z_b) + \rho_w g z_b = \rho_w g h \tag{5}$$

4.4 SHAKTI model

Many subglacial hydrology models have been developed and are used with some success, although widespread use coupled to ice dynamics models remains a challenge. For an overview and summary of model development, Flowers, 2015 and de Fleurian et al., 2018 are helpful resources. As one example of a subglacial hydrology model, SHAKTI (Subglacial Hydrology And Kinetic, Transient Interactions; A. Sommers et al., 2018) uses a single set of equations to calculate hydraulic head (from which effective pressure is easily obtained), basal water

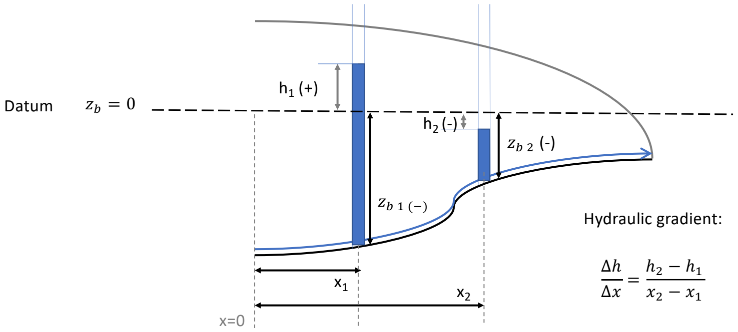

Figure 7: Hydraulic head and hydraulic gradient relative to an arbitrary datum.

Table 1: Variables used in SHAKTI model

Symbol	**Units**	**Description**
b	m	Subglacial gap height (average over element)
h	m	Hydraulic head
K	$m^2\ s^{-1}$	Hydraulic transmissivity, $K = b^3 g/(12\nu(1 + \omega Re))$
$\dot{m}$	$kg\ m^{-2}\ s^{-1}$	Subglacial melt rate
N	Pa	Effective pressure, $N = p_i - p_w$
p_i	Pa	Ice overburden pressure, $p_i = \rho_i g H$
p_w	Pa	Subglacial water pressure $p_w = \rho_w g(h - z_b)$
$\mathbf{q}$	$m^2\ s^{-1}$	Gap-integrated water flux
Re	Dimensionless	Reynolds number, Re=$\lvert q\rvert/\nu$
t	s	Time
β	Dimensionless	Parameter controlling opening due to sliding over bed bumps, $\beta = (b_r - b)/l_r$ for $b < b_r$, $\beta = 0$ for $\beta \geq b_r$
$\boldsymbol{\tau}_b$	Pa	Basal stress

flux, and geometry of the subglacial drainage system. In contrast to many other subglacial hydrology models, SHAKTI facilitates natural transitions between laminar and turbulent flow, allowing different regimes to coexist in different regions of the model domain. SHAKTI includes heat generated energy dissipation from the flow and opening by melt across the entire domain, unlike other models that treat “inefficient” sheet-like and “efficient” channel-like components of the drainage system with different equations. Following is a summary of the model equations.

4.4.1 Model equations

SHAKTI is composed of a system of equations to describe conservation of ice and water mass, drainage configuration, water flux, and internal melt generation. Tables 1 and 2 define variables, constants, and parameters used in the model.

Table 2: Constants and parameters used in SHAKTI model

Symbol	**Value**	**Units**	**Description**
A	4.9×10^{-25}	$\mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$	Flow law parameter
b_r	0.1	m	Typical height of bed bumps
c_t	7.5×10^{-8}	$\mathrm{K\ Pa}^{-1}$	Change of pressure melting temperature
c_w	4.22×10^{3}	$\mathrm{J\ kg}^{-1}\ \mathrm{K}^{-1}$	Heat capacity of water
G	0.05	$\mathrm{W\ m}^{-2}$	Geothermal flux
g	9.81	$\mathrm{m\ s}^{-2}$	Gravitational acceleration
$i_{e\to b}$	Varying (prescribed)	$\mathrm{m\ s}^{-1}$ or $\mathrm{m}^3\ \mathrm{s}^{-1}$	Input rate of meltwater
L	3.34×10^{5}	$\mathrm{J\ kg}^{-1}$	Latent heat of fusion of water
l_r	2.0	m	Typical spacing between bed bumps
n	3	Dimensionless	Flow law exponent
$\mathbf{u}_b$	Varying (prescribed)	$\mathrm{m\ s}^{-1}$	Sliding velocity
ν	1.787×10^{-6}	$\mathrm{m}^2\ \mathrm{s}^{-1}$	Kinematic viscosity of water
ω	0.001	Dimensionless	Controls laminar/turbulent transition
ρ_i	917	$\mathrm{kg\ m}^{-3}$	Bulk density of ice
ρ_w	1000	$\mathrm{kg\ m}^{-3}$	Bulk density of water

The water balance equation is written as:

$$\frac{\partial b}{\partial t} + \nabla \cdot \mathbf{q} = \frac{\dot{m}}{\rho_w} + i_{e\to b} \tag{6}$$

where b is the gap height between the ice and bed (units of m), $\mathbf{q}$ is gap-integrated water flux through the subglacial system (units of $\mathrm{m}^2\ \mathrm{s}^{-1}$), $\dot{m}$ is the melt rate (units of $\mathrm{kg\ m}^{-2}\ \mathrm{s}^{-1}$), ρ_w is density of water (1000 $\mathrm{kg\ m}^{-3}$), and $i_{e\to b}$ is the rate of meltwater input from the englacial system to the bed, which can be specified and handled by the model as a combination of distributed input (units of $\mathrm{m\ s}^{-1}$) and point inputs to represent moulins or crevasses (units of $\mathrm{m}^3\ \mathrm{s}^{-1}$).

The configuration of the drainage system is represented by the average gap height b over an element, which evolves through time dynamically. Gap height increases by melt and by sliding over bumps in the bed and decreases due to creep deformation. This can be expressed as change in gap height over time:

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} + \beta\,|\mathbf{u}_b| - A|p_i - p_w|^{n-1}(p_i - p_w)b \tag{7}$$

where ρ_i is the density of ice (917 $\mathrm{kg\ m}^{-3}$), β is a dimensionless coefficient that dictates opening of the subglacial gap by sliding over bumps in the bed, u_b is the sliding velocity (units of $\mathrm{m\ s}^{-1}$), A is the flow law parameter (units of $\mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$), $p_i = \rho_i g H$ is ice overburden pressure (units of Pa) where g is gravitational acceleration and H is ice thickness, $p_w = \rho_w g(h - z_b)$ is subglacial water pressure (units of Pa) where h is hydraulic head (units of m) and z_b is bed elevation above sea level (units of m), and n is the flow law exponent (unitless).

The momentum equation that describes the water flux is:

$$\mathbf{q} = \frac{-b^3 g}{12\nu(1+\omega Re)}\nabla h \tag{8}$$

where ν is kinematic viscosity of water ($1.787 \times 10^{-}6$ m^2 s^{-1}), ω is a parameter that controls the transition from laminar to turbulent flow, and Re is the local Reynolds number. When $\omega Re << 1$, Eqn. (3) behaves like laminar flow, $\mathbf{q}\ \nabla \mathbf{h}$. When $\omega Re >> 1$, $\mathbf{q}\ \sqrt{\nabla \mathbf{h}}$, as in completely turbulent flow.

Internal melt generation is represented in SHAKTI through an energy balance at the bed, assuming ice and water are both always at the pressure melting point temperature:

$$\dot{m} = \frac{1}{L}\left(G + |\mathbf{u}_b \cdot \boldsymbol{\tau}_b| - \rho_w g\ \mathbf{q} \cdot \nabla h + c_t c_w \rho_w \mathbf{q} \cdot \nabla p_w\right) \tag{9}$$

where L is the latent heat of fusion of water (3.34×10^5 J kg^{-1}), G is geothermal heat flux (units of W m^{-2}), τ_b is the basal stress (units of Pa), c_t is the change of pressure melting point temperature with pressure (7.5×10^{-8} kg Pa^{-1}), c_w is the heat heat capacity of water (4.22×10^3 J kg^{-1} K^{-1}). This energy equation includes melt due to heat contributed by geothermal sources, friction from sliding, turbulent dissipation, and adjustments for changes in the pressure melting point due to changes in the water pressure.

Equations (6)-(9) are combined to form an elliptic equation in terms of hydraulic head:

$$\nabla \cdot \left(\frac{-b^3 g}{12\nu(1+\omega Re)}\nabla h\right) = \dot{m}\left(\frac{1}{\rho_w} - \frac{1}{\rho_i}\right) + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta\,|\mathbf{u}_b| + i_{e\to b} \tag{10}$$

4.4.2 Numerical methods

Equation (10) is solved for the head distribution using an iterative approach to handle the nonlinear dependence of the terms on the right-hand side of the equation, then Equation (7) is solved explicitly to evolve the gap height. SHAKTI is built into the Ice-sheet and Sea-level System Model (ISSM, https://issm.jpl.nasa.gov/; Larour et al., 2012) using finite element methods in a highly parallelized computational framework. For more details, please see A. Sommers et al., 2018 or A. N. Sommers, 2018.

5 Planetary glacier hydrology

- Geysers on Enceladus (Nimmo et al., 2007, work in progress by Meyer et al.)
- Nitrogen glaciers on Pluto (Howard et al., 2017)
- Liquid water beneath Martian ice caps? Or something else? (Schroeder and Steinbrügge, 2021)

- etc.

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