Satellite Gravimetry and its Application to Glaciology
by Anthony Arendt for the UAF Summer School in Glaciology, June 2010

1 Overview

The Earth is a dynamic system in which components of the core, surface and atmosphere are mobile and redistribute mass on various time scales. These processes produce variations in Earth's gravitational field. Until recently, the gravity field has been treated as static, and most efforts have focused on developing terrestrial, airborne and and spaceborne methods for improved mapping of this field. Advances in satellite ranging technology and GPS are providing new ways to map the small-scale departures of the gravity field caused by processes that vary on time scales from hours to thousands of years. Geophysical processes observable on timescales of a typical satellite mission include post-glacial rebound, changes in sea level, redistribution of atmospheric mass, and changes in the cryosphere (Fig. 1). The goal of this document is to explain the basic equations of gravity and show how recent satellite measurements are processed to construct time-variations in gravity for glaciological studies.

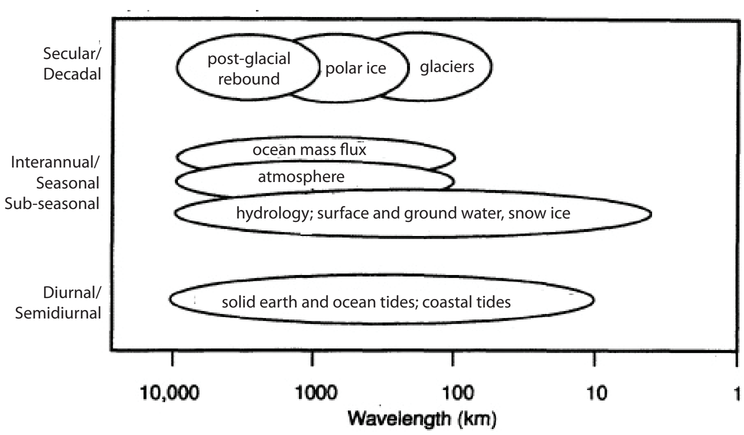


Figure 1: Geophysical phenomena that cause measurable temporal and spatial variation in the Earth's gravity field. Adapted from Bettadpur and Tapley (1996).

2 Theory

According to Newton's law of gravitation, the force F of gravitation is inversely proportional to the inverse square distance l between the two masses. Let m be the attracting mass, and setting the attracted mass equal to unity

$$F = G\frac{m}{l^2} \tag{1}$$

where G is Newton's gravitational constant ($6.6742 \cdot 10^{-11} \mathrm{m^3 kg^{-1} s{-}2}$).

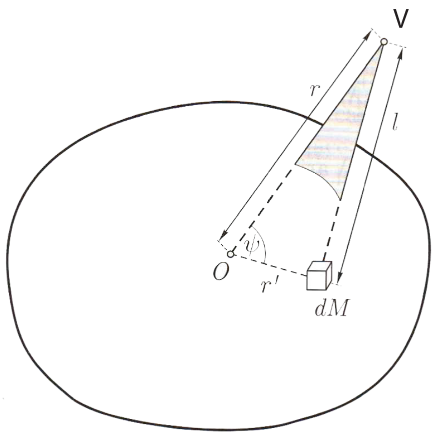


Figure 2: The potential V at a point a distance l from an attracting mass is the summation of the potential due to all mass elements dM. Adapted from Hofmann-Wellenhof and Moritz (2006).

We introduce a scalar field called the gravitational potential (or the geopotential)

$$V = \frac{Gm}{l} \tag{2}$$

that is related to the gravity force vector as

$$\mathbf{F} = \nabla V \tag{3}$$

This shows that the derivative of the geopotential along a specific path is equal to the vector force of gravity in the direction of that path. The advantage of this formulation is that in geodesy it is easier to deal with a single function (the gravitational potential) rather than the three components of the gravitational force.
When calculating the effects of Earth's gravity on a satellite we need to solve for the geopotential at a specific location outside of the Earth. To do this we integrate the geopotential over a series of mass elements dM that comprise the Earth (Fig. 2)

$$V = G \iiint_{Earth} \frac{dM}{l} \tag{4}$$

At a point outside of any attracting mass the gravitational potential is a harmonic function and satisfies Laplace's equation

$$\Delta V = 0 \tag{5}$$

Therefore the potential can be expanded into a series of spherical harmonic functions of degree l and order m

$$V(r, \vartheta, \lambda) = \frac{GM}{r} \sum_{l=0}^{\infty} \left(\frac{R}{r}\right)^{l+1} \sum_{m=0}^{l} \bar{P}_{lm}(\sin\vartheta)(\bar{C}_{lm}\cos(m,\lambda) + \bar{S}_{lm}\sin(m,\lambda)) \tag{6}$$

where r, ϑ, λ are the spherical geocentric radius, latitude and longitude coordinates of the point where the gravitational potential is calculated; R is the Earth's mean semi-major axis; $\bar{C}$ and $\bar{S}$ are dimensionless Stokes Coefficients, the overbar indicating these are "fully normalized" *; and $\bar{P}$ is the fully normalized associated Legendre polynomial, which are a series of polynomials that have been shown to satisfy Laplace's equation.
Spherical harmonics come in a variety of different forms, but the essential idea is that they comprise a superposition of orthogonal sine and cosine functions of varying wavelengths that can be used to represent a particular field on a sphere. As you progress to higher degree and order terms you achieve higher resolution in representing the field on the sphere.
The term $\left(\frac{R}{r}\right)^{l+1}$ in Equation 6 is the attenuation factor, showing that the gravity signal becomes increasingly attenuated with increasing distance r from the attracting mass. Note that this attenuation is wavelength dependent, such that with increased separation the shorter wavelengths become attenuated more than longer wavelengths. This illustrates a fundamental problem in satellite gravimetry: lower satellite orbits result in less attenuation, but also subject the satellite to additional atmospheric frictional forces.

Next, it is useful to express the Earth's gravity field in terms of the shape of the geoid N (Chao et al., 1987)

$$N(\vartheta,\lambda) = R\sum_{l=0}^{\infty}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\vartheta)(\bar{C}_{lm}\cos(m,\lambda)+\bar{S}_{lm}\sin(m,\lambda)) \tag{7}$$

The above equation describes the static gravity field, but for glaciological applications we are interested in time variations in Earth gravity. The change in N can be related to variations $\Delta\bar{C}_{lm}$ and $\Delta\bar{S}_{lm}$ in the Stokes coefficients

$$\Delta N(\vartheta,\lambda) = R\sum_{l=0}^{\infty}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\vartheta)(\Delta\bar{C}_{lm}\cos(m,\lambda)+\Delta\bar{S}_{lm}\sin(m,\lambda)) \tag{8}$$

Next we relate a local change in surface mass density $\Delta\sigma(\vartheta,\lambda)$ to the $\Delta\bar{C}_{lm}$ and $\Delta\bar{S}_{lm}$ (Chao et al., 1987; Swenson and Wahr, 2002)

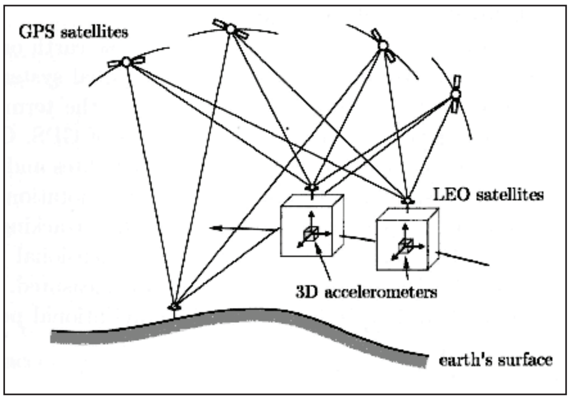
$$\Delta\sigma(\vartheta,\lambda) = \frac{R\rho_E}{3}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\frac{2l+1}{1+k_l}\bar{P}_{lm}(\cos\vartheta)(\Delta\bar{C}_{lm}\cos(m,\lambda)+\Delta\bar{S}_{lm}\sin(m,\lambda)) \tag{9}$$

where ρ_E is the average density of the solid Earth and k_l are the load love numbers (see Wahr et al. (1998)). This equation recognizes two components of the change in density $\Delta\sigma$, namely the change due to the added surface density assuming a rigid Earth, as well as the resultant elastic yielding of the Earth that tends to counteract the additional surface density. The latter component explains why load love numbers come into this equation.

*Fully normalized spherical harmonics are mathematically easier to handle. They are normalized such that the average square of any fully normalized harmonic is unity.

2.1 Satellite Gravimetry

Satellite gravimetry measures the gravitational potential affecting an orbiting body in order to reconstruct variations of the geoid. There are three key requirements in any satellite gravimetry mission: uninturrepted tracking of the satellite's three-dimensional position; measurement or compensation for non-gravitational forces; and achieving a low Earth orbit. Uninturrepted tracking is necessary in order to fully map the Earth's gravity on the satellite orbit and to overcome resonances that are set up resulting from each satellite's particular orbital frequency. Early orbital analysis tracked satellites from the ground and only sampled a small portion of their orbit. These early missions also treated the satellite as a test mass in free fall in Earth's gravitational field, however we know that a satellite is affected not only by Earth's gravity but also by non-gravitational accelerations due to atmospheric drag and solar radiation forcing. Therefore it is important either to measure or to correct for non-gravitational forces acting on the satellite. Finally, a low orbit is necessary to minimize signal attenuation. Note that the requirement of a low orbit is in conflict with the need for accurate accounting of non-gravitational effects, because atmospheric drag increases as the orbital height decreases.
Modern satellite missions use gravity gradiometry, mapping variations in gravitational acceleration, to determine higher order terms in the geopotential that helps overcome the effects of attenuation. The most recent gravity mission is the Gravity field and steady-state Ocean Circulation Explorer (GOCE), launched on March 2009 and consisting of a single Low Earth Orbit (250 km) satellite. This satellite is equipped with a 3-dimensional gradiometer to measure all components of geopotential variations over very short (50 cm) baselines.



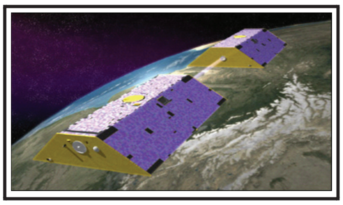


Figure 3: The Gravity Recovery and Climate Experiment is equivalent to a gradiometer in one dimension with a long (220 km) baseline.

The Gravity Recovery and Climate Experiment (GRACE) was launched March 2002 and is a joint mission between NASA and the German Aerospace Center (DLR) (Fig. 3). It consists of a pair of

Low Earth Orbiting satellites (about 500 km above the Earth) separated by about 220 km. Ranges and range rates are measured using a highly precise K-band radar,and GPS receivers provide precise positioning of the satellites. Accelerometers are used to calculate the effects of atmospheric drag and solar radiation pressure. The GRACE satellites were launched in March 2002 and have already exceeded their expected 5-year lifespan. Eventually their orbits will decay to the extent that atmospheric drag and other errors will become too large for accurate monitoring of the gravity field. In this document we focus on the GRACE satellites because they are optimized for mapping the static gravity field which is of interest for glaciological studies.

3 GRACE Data

GRACE data are distributed in two main formats, a Level 1 product that contains among other parameters the K-band inter-satellite range and range-rate (KBRR) variations, and a Level 2 product containing monthly Stokes coefficients and error estimates on these coefficients, from which one can generate the mean gravity field. There are numerous processing centers but the primary data access portal is `http://podaac.jpl.nasa.gov/grace`. The University of Colorado at Boulder has a useful site for quick visualization of GRACE data series (`http://geoid.colorado.edu/grace`).

In the literature you will find two main approaches for taking GRACE data (either Level 1 or the Level 2 product) and determining mass changes of a specific geophysical field over a specified geographic region. The "spherical harmonic" approach begins with the Level 2 product, averages the calculated geoid changes over a region of interest, and corrects for the effects of signal leakage and contamination from other geophysical parameters. The "mascon" approach involves a direct calculation of the change in the Stokes coefficients through knowledge of the satellite orbital parameters (i.e. the Level 1 product) and the response of those parameters to some fixed change in the Stokes coefficient.

3.1 Spherical Harmonic Method

The Level 2 product contains dimensionless fully normalized Stokes coefficients $\bar{C}_{lm}$, $\bar{S}_{lm}$ that characterize the mean gravity field over a 30 day period. From Equation 9 one can easily calculate the local change in mass density resulting from changes in these coefficients between months. However, we now run in to an important limitation of GRACE measurements, namely that the satellite measurement errors increase rapidly with increasing degree l. In other words, GRACE can only "see" changes in mass up to a limited resolution, so that it is necessary to truncate our spherical harmonic expansions at a specific threshold. Most studies truncate their GRACE solutions at degree 100, corresponding to a length scale that can be approximated by the relation 20,000 km$/l = 200$ km or larger. By truncating our solution at a threshold degree the relation in Equation 8 is no longer equivalent to a point measurement. This problem is avoided by averaging Equation 8 over a region.

3.1.1 Spatial Averaging of GRACE Data

Plotting GRACE Level 2 data is a useful exercise for visualizing spatial and temporal variations in the geoid, but using the data for calculating mass changes of specific regions requires additional

processing. The most common method is to apply a spatial averaging filter to the spherical harmonic representation of the geoid change. This helps to minimize satellite errors as well as the degradation of solution quality at high degrees.
Spatial averaging involves defining an averaging kernel φ that describes the shape of your particular region of interest (in our case, an ice sheet or region of mountain glaciers)

$$\varphi(\vartheta,\lambda) = \begin{cases} 0 & \texttt{outside the basin} \\ 1 & \texttt{inside the basin} \end{cases} \tag{10}$$

The change in vertically integrated water storage averaged over an arbitrary region is then simply the summation of Equation 8 over that portion of the sphere described by the region of interest in Equation 10.
There are numerous ways in which this averaging procedure can be refined and optimized. Equation 10 describes an exact averaging kernel, which has problems of ringing near the boundaries of the basin mask (the so-called "Gibbs" phenomenon). One way around this is to use an approximate rather than exact averaging kernel, the most common of which is a Gaussian filter. This creates a smooth transition of mass across the basin boundary, but inevitably results in sampling of mass that is outside the region of interest. The difference between the exact and approximate summation of mass over the region is defined as "leakage". Most studies carry out an extensive series of tests to optimize the trade-off between spatial resolution, signal degradation at the basin boundary and signal leakage.

3.1.2 Removal of Contamination from other Geophysical Sources

Various geophysical phenomena producing secular and non-secular signals in the geoid within the lifespan of a typical satellite gravity mission. Variations in glaciers and ice sheets have potentially large effects on the secular geoid rates, and it is important to note that these variations fall within the same magnitude range of post-glacial rebound (Fig. 4). Glaciers and ice sheets also produce signals in the geoid amplitude that are right within the range of ocean, atmosphere and hydrology variations (Fig. 5). Because cryospheric signals overlap with several other geophysical processes on secular and seasonal timescales, one of the most important steps in the processing of GRACE data is the removal of these other sources of mass variability (Fig. 6).
The GRACE processing team provides additional Stoke's Coefficients as part of the Level 2 product that describe variations in mass due to atmospheric variability and ocean tides. These are determined for example using the ECMWF reanalysis products for the atmospheric mass, and one of several ocean tide models for the ocean mass component. It is then easy to remove those components prior to the averaging of Level 2 data to determine mass trends. However, terrestrial hydrology and PGR signals are not calculated by the GRACE processing team, and must be generated independently using one of several available global models. An important consideration is that while global PGR models account for the Earth's response to long-term glacier changes, for example the collapse of the Laurentide ice sheet and the subsequent uplift of the Hudson's Bay region, they do not consider local changes in response to recent deglaciation events. A good example is the collapse of the Glacier Bay icefield in Alaska that began at the end of the Little Ice Age. The rapid retreat of this tidewater system has produced some of the largest rates of uplift on Earth, and such a signal contributes to about 10% of the observed mass change in the Gulf of Alaska (Arendt et al., 2008).

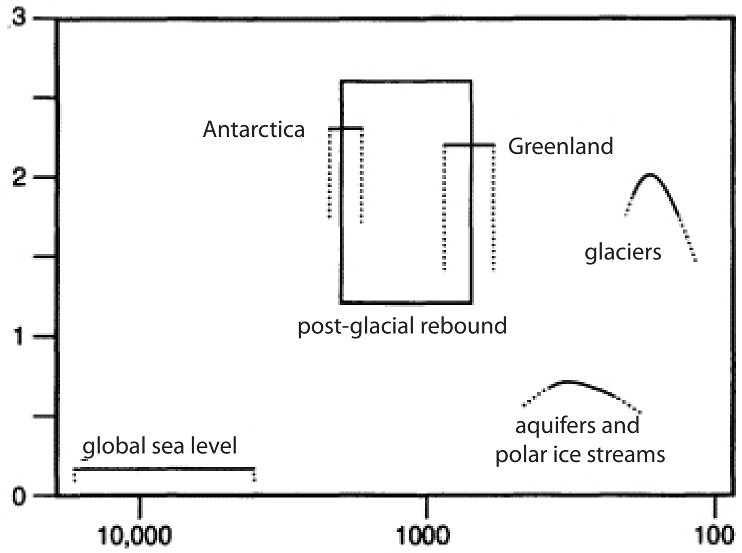


Figure 4: Geophysical fields that vary approximately linearly over the lifespan of a gravity mission. Adapted from Hofmann-Wellenhof and Moritz (2006).

3.2 mascon Method

The history of this approach goes back to early NASA studies of the effects of lunar gravity anomalies on orbiting satellites. Observed distortions in early lunar orbiters led to the discovery of surface gravitational anomalies, spurring additional research to correct for the effect of these anomalies and provide for more precise landing of lunar spacecraft. A region of positive gravitational anomaly was termed a “mass concentration” or “mascon” for short.

Understanding the effects of gravitation anomalies on satellite orbits became a useful tool in the development of GRACE processing techniques. From this work the Space Geodesy Laboratory at the NASA Goddard Space Flight Center developed a set of procedures to calculate changes in mass at specific blocks or grid cells that they termed “mascons”. The mathematics of this approach follow the theory laid out above, but rather than solving for the change in mass from changes in Stokes coefficients, Equation 9 is rearranged so that we express the Stokes coefficients as the independent variables

$$\Delta\bar{C}_{lm}(t) = \frac{(1+k_l)\,R^2\Delta\sigma(\vartheta,\lambda)}{(2l+1)\,M}\int \bar{P}_{lm}(\cos\vartheta)\cos(m,\lambda)d\Omega \tag{11}$$

$$\Delta\bar{S}_{lm}(t) = \frac{(1+k_l)\,R^2\Delta\sigma(\vartheta,\lambda)}{(2l+1)\,M}\int \bar{P}_{lm}(\cos\vartheta)\sin(m,\lambda)d\Omega \tag{12}$$

Next, a series of mascons are defined to outline the region of interest. Various dimensions have been tested, but to date the fundamental resolution of GRACE observations has been shown to be approximately 2x2 equal area degrees. For each mascon, $\sigma(\vartheta,\lambda)$ is given a value of 1 cm of water over the region of interest. One then calculates the response of the differental Stokes coefficients to this arbitrary addition of mass. The next step is to determine the response of *i* GRACE satellite

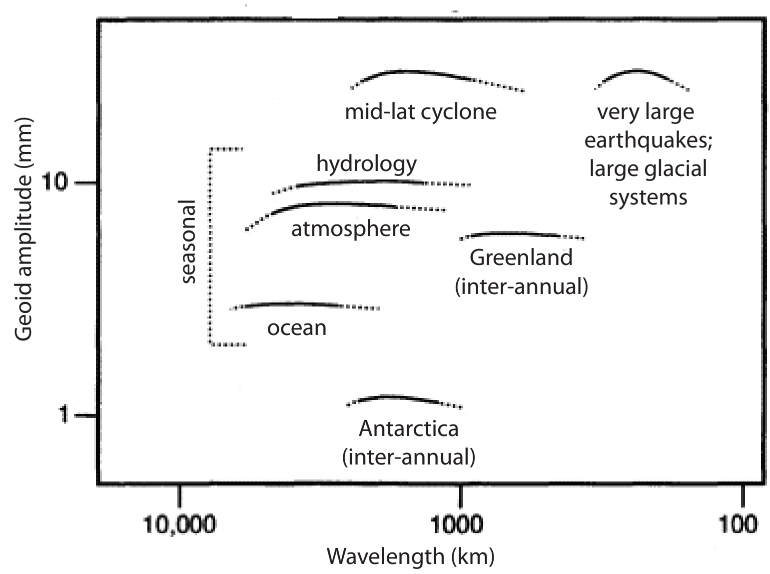


Figure 5: Geophysical fields that do not vary linearly over the lifespan of a gravity mission. Adapted from Hofmann-Wellenhof and Moritz (2006).

observation O (as characterized by a series of KBRR observations) to changes in Earth mass as characterized by changes in the Stokes coefficients. We then have two partial derivatives from which one calculates a series of j mascon parameters P_j

$$\frac{\delta O_i}{\delta P_t^j} = \sum_{l=1}^{l_{max}} \sum_{m=0}^{l} \frac{\delta O_i}{\delta \bar{C}_{lm}} \cdot \Delta \bar{C}_{lm}^j + \frac{\delta O_i}{\delta \bar{S}_{lm}} \cdot \Delta \bar{S}_{lm}^j \tag{13}$$

The resulting mascon parameters P_j are a set of scaling parameters that are applied to the set of differential Stokes coefficients calculated in response to the loading of 1 cm of water. These parameters are calculated relative to the mean field as well as a series of forward models of other geophysical processes that contaminate the solution.

4 Calculating Mass Balance from a GRACE Time Series

Regardless of the solution procedure chosen, one eventually arrives at a time series that best represents the geophysical signal of interest, which is in our case the glacier mass balance. All GRACE solutions result in a time series of mass, usually in units of gigatons (Gt), representing the mass of the Earth relative to the mean background gravity field. The magnitude of published GRACE solutions are therefore arbitrary, and certainly do not tell us anything about the total mass of the glacier system.

Because multiple passes of the GRACE satellites are required to acquire an accurate determination of the geoid, mass estimates from GRACE represent temporally-averaged quantities. Therefore the GRACE data series are not directly comparable with conventional mass balance measurements that generally involve the subtraction of a series of instantaneous measurements (for example, mea-

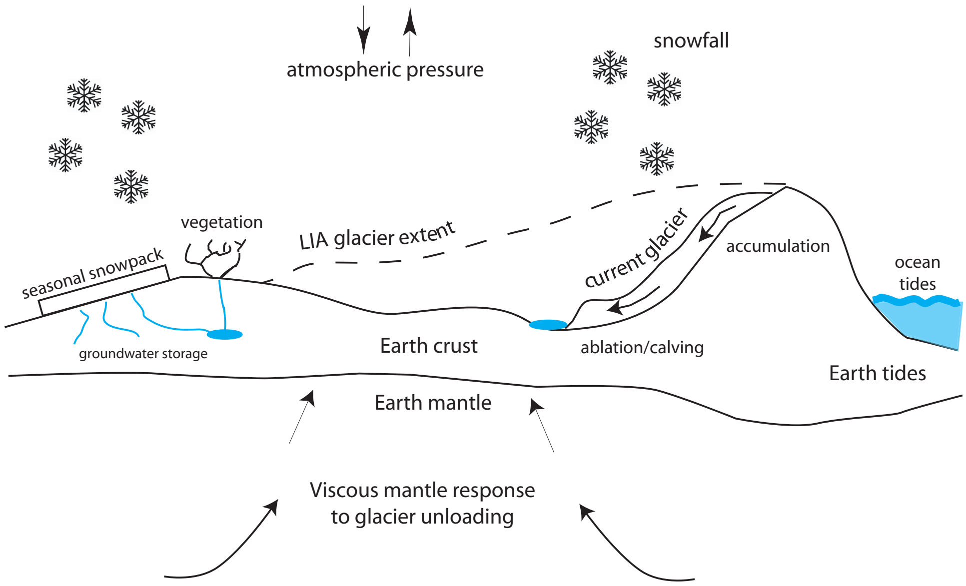


Figure 6: Conceptual diagram of geophysical processes contributing to mass variations sensed by satellite gravimetry.

surements an ablation stake or snow pit). Temporal averaging of ground measurements or model outputs is required when comparing hydrological data to GRACE observations (see Swenson and Wahr (2006) for specific equations). This is particularly important for modeling studies that simulate mass balances at time scales at or below the temporal resolution of the GRACE solution (e.g. Arendt et al. (2009)). For calculation of seasonal changes where the long time period between measurements minimizes these temporal averaging errors, this problem is often ignored.
The trend and amplitude of a GRACE time series are the most relevant terms for mass balance analysis (Fig. 7). The trend is usually calculated as a rate per year, in which case this describes the time-averaged mass balance of the system of glaciers in the study domain. The amplitude is directly comparable to the “mass balance amplitude” described by Meier (1984), also termed the “rate of mass turnover”. Both of these calculations, in particular the trend, are often done incorrectly in the literature. It is important that any calculation of trend be done on a GRACE solution that spans an integer number of years. Otherwise, the trend will be biased by the inclusion of an additional melt or accumulation season. Also it is necessary to isolate the periodic component from the long term trend. For example, a linear fit of the GRACE solution without removing the periodic component(s) first may bias the trend based on the structure of the periodicity.

5 Summary Comments

The application of satellite gravimetry to cryospheric studies is still in its early stages, and much additional study is necessary. Research on the mass balance of ice sheets and glaciers as determined from GRACE vary in some cases by 100% or more, even when considering similar time periods. One reason for these discrepancies are the number of different methods that have been developed for processing GRACE data. Selecting a subregion of a global spherical harmonic expression of

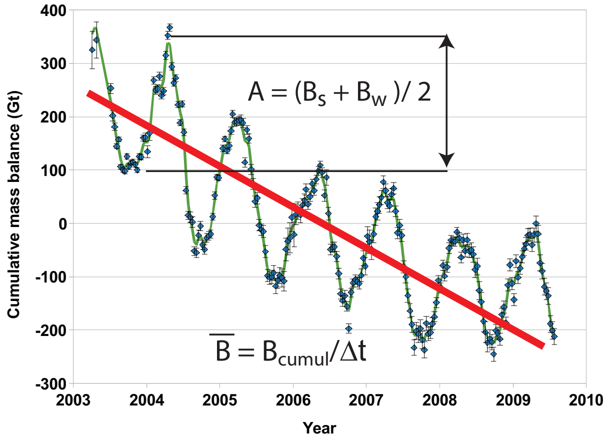


Figure 7: Sample GRACE time series showing cumulative ($\bar{B}$) and summer/winter (B_s, B_w) mass balances. A is the mass balance amplitude.

geoid changes is the most common approach, but there are numerous choices to make when applying smoothing algorithms developing an averaging kernel. The mascon approach uses short-arc data reduction to isolate gravity changes occurring only over the region of interest, and relies only on the intersatellite range/range rate observations to determine changes in the geoid. However this approach also involves the application of spatial constraint equations that introduce some ambiguity similar to what occurs when smoothing the spherical harmonic solutions. Both spherical harmonic and mascon approaches require corrections for non-glaciological sources of mass change, and in many regions there is a lack of model or observational data to provide these corrections. Future efforts will focus on better accounting for the non-glaciological background signals and on combining GRACE with other remote sensing platforms to help constrain that component of the signal related to cryospheric changes.

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