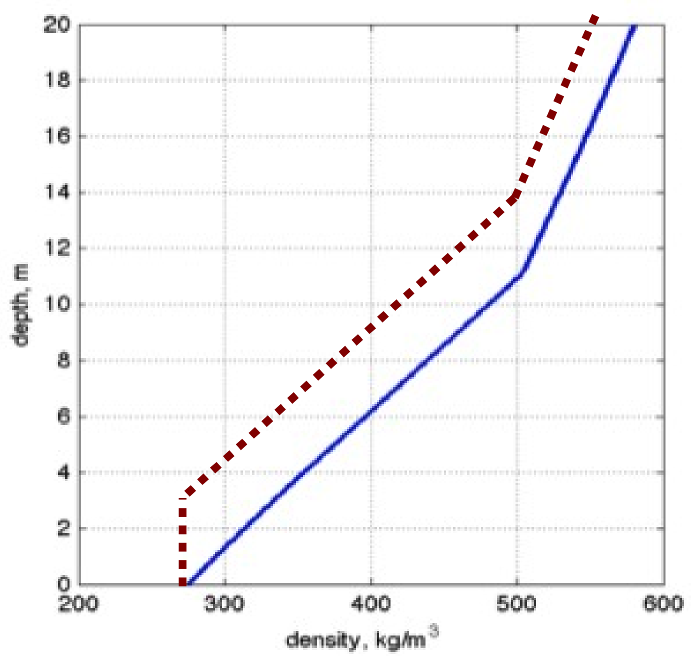
1a. After the storm, there is a layer of fresh snow on the surface. Its thickness is found from its mass per unit area as:

$$h_{layer} = \frac{m_{layer}}{\rho}$$

Its mass per unit area is 917 kg /m^2 , so its thickness is

$$h_{layer} = \frac{917\,kg}{m^2}\,x\,\frac{m^3}{280\,kg} = 3.3\,m$$

This displaces the density curve downwards by 3.3 m:



1b.

The rate of change in the layer thickness is found by differentiating the above equation with respect to time, using the chain rule.

$$\frac{dh_{layer}}{dt} = -\frac{1}{\rho} h_{layer} \frac{d\,\rho}{dt}$$

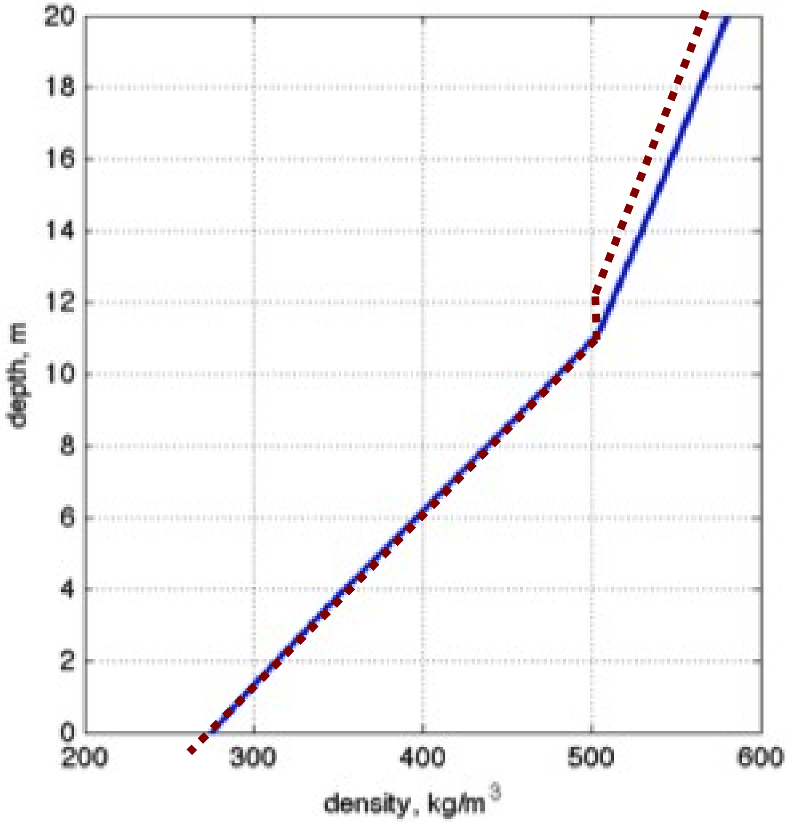
or

$$\frac{dh_{layer}}{dt} = -\frac{1}{\rho} h_{layer} F(\rho)$$

Values of F for t=0 and t=50 years can be found from the given plots for the steady-state firn column. At t=0, the density is equal to the surface density, and the densification rate is approximately 7 kg m^{-3} yr^{-1}. At t=50 years, the steady-state depth is around 11.5 m, slightly past the kink in the density profile. The corresponding density is around 550 kg m^{-3}, so the densification rate is about 1.8 kg m^{-3} yr^{-1}. This gives a thinning rate of 5 mm/yr.

1c.

No additional assumptions are needed to give the 50-year density profile. The layer thickness is 1.6 m, and its top is at 11.5 m (approximately)



1d

If there were a 10-year hiatus, the firn column would continue to age, following the trajectory shown, but without more snow being added to the top of the profile. This would be equivalent to shifting the entire density profile to shallower depth. The amount of this shift can be found from the depth-age scale: an age of 10 years corresponds to a depth of about 3 meters. On the surface, a thinning rate of about 30 cm /year would be observed.

1e.

The lower-density layer would have a thickness of

$$h_{layer} = \frac{91.7\, kg}{m^2} x \frac{m^3}{150\, kg} = 0.61\, m$$

This would displace the density profile downwards by 0.61 m. Extrapolating the densification rate to 150 kg m^{-3} gives about 8 kg m^{-3} yr^{-1} for the layer when it's at the surface. This gives a thinning rate of 3.2 cm/yr at the surface.

At a constant densification rate of 8 kg m^{-3} yr^{-1}, it would take the layer about 18 years to densify to 300 kg m^{-3}, the first density for which we have a densification rate on the plot. Between 18 and 50 years, the densification would proceed as it does for snow that falls on the surface at 300 kg m^{-3}. This lets us read the density of the layer after 50 years from the age-depth and density-depth plots (at 32 years, corresponding to 7 meters and 450 kg m^{-3}), and determine the densification rate: 5 kg m^{-3} yr^{-1}. This implies a thinning rate of 2.3 mm/yr.

2a.

Assume a near-surface density of 300 kg m^{-3}.

$$\frac{250\,kg}{m^2\,yr}\,x\,\frac{40}{100}\,x\,\frac{m^3}{300\,kg}=0.33\,\frac{m}{yr}$$

2b.

The error in the slope is approximately

$$\sigma_{slope}=\frac{\sigma_h}{\sigma_y}\frac{1}{(N-1)^{1/2}}$$

where N is the number of years. This comes to a slope of 0.0015, or 0.8 degrees.

2c.

The error introduced in the elevations is approximately $\sigma_{slope}\sigma_y$, or 0.15 m. The geolocation error is $8\,m\times\pi/180$ =0.14 m.

3a. The mass-balance rate is the difference between the upstream accumulation and the downstream discharge. The upstream accumulation is +200 kg $m^{-2}yr^{-1}$ x $2.03x10^{10}$ m^2, or 4.06 Gt/yr

There is a range of possible discharge values, because the mechanism of ice motion is not specified. If it is frozen to the bed, the discharge flux could be as little as 80% of the product of the ice thickness and the surface velocity; if all motion is by sliding, it could be 100%.

This gives a range of discharge values between 4.5 Gt/yr (for 100% sliding) and 3.6 Gt/yr (for zero sliding). The range of mass rates for the upstream region is between -0.44 Gt/yr and +0.46 Gt/yr

3b. The total mass-balance rate for the glacier is the sum of that of the upstream region and that of the trunk. Assuming that the trunk discharge is equal to the upstream flux into the trunk, the trunk surface mass balance contributes $9x10^8 m^2$ x -150 kg $m^{-2}yr^{-1}$ =-0.135 Gt/yr to the net mass balance, giving a total between -0.58 and 0.33 Gt/yr.

3c. Assuming steady state, the output flux is the sum of the input flux and the surface mass balance for the downstream region. This gives discharge rates between 4.36 and 3.47 Gt/yr. Dividing by the gate width, the mean surface speed, and the density of ice gives the thickness and, for simplicity, I assume that the outlet motion is dominated by sliding. This gives thicknesses between 408 and 515 m.

3d. The glacier's net mass balance is the sum of the surface mass balance values and the discharge flux. The total surface mass balance is +4.06 Gt/yr for the upstream region, and -.135 Gt/yr for the downstream region, for a total of +3.93 Gt/yr. The output flux is 1250 m yr^{-1} x 10^4m width x the density of ice times the ice thickness, or between -5.9 and -4.7 Gt/yr, giving a basin net balance between -2 and -0.8 Gt/yr.

3e. If the downstream region was thinning at 0.5 m/yr in the downstream region, the output flux estimate changes, the ice outflow must have included the ice lost in lowering the surface, so more flux out is needed to balance the input flux. A thinning rate of 0.5 m/yr corresponds to a mass balance rate of

-0.5 m/yr x 917 kg/cubic meter 900 square kilometers = -0.41 Gt/yr.

This leads to discharge estimates for year 1 between -3.88 and -4.77 Gt/yr, with thickness estimates between 458 and 563 m.

Since the thinning at the front was 2 m/yr, the front thickness at the start of year 2 would have been between 456 and 455 m. Multiplied by the surface speed, the glacier width, and the density of ice, this gives year-2 discharges between -5.24 and -6.45 Gt/yr, with total mass-balance rates between -2.51 and -1.31 Gt/yr.

4a. The pressure at the bottom of the ice column is $-\rho_w g z_{bottom}$, while the glaciostatic pressure is $\rho g (z_{surface} - z_{bottom})$, where all z values are relative to sea level, and all pressures are gauge pressures (not including atmospheric pressure).

This gives an equation for the hydrostatic equilibrium:

$$\rho g (z_{surface} - z_{bottom}) = -\rho_w g z_{bottom}$$

If the firn column is made up of h_{air} meters of air and $h - h_{air}$ meters of ice, then $h\rho = \rho_{ice}(h - h_{air})$.

Substituting this into the hydrostatic equation, with $h = z_{surface} - z_{bottom}$ gives

$$\rho_{ice}(h - h_{air}) = \rho_w z_{bottom}$$

or

$$z_{surface} = h - \frac{\rho_{ice}}{\rho_w}(h - h_{air})$$

4b. The fractional air density of the firn column is 1-902 kg/m^3 / 917 kg/m^3 = 0.0164, so the initial h_{air} is 0.0164x800 m or 13.1m. This gives an initial surface height of 99.4 m.

Each cubic meter of the additional layer is made up of 381 m^3 of ice and 618 m^3 of air. This means that h increases by 1 meter, and h_{air} increases by 0.618 m. This gives a final surface height of 100.1 m, and a height difference of +.66 m.

4c. The near-grounding-line melt rate is likely larger than the melt rate at 5 km. This means that the minimum melt rate estimate for the region between 0 and 5 km downstream of the grounding line is 4 m/yr. The ice takes 5 years to traverse this distance, so the minimum contribution of melting in the region between 0 and 5 km is -20 m of ice.

4d. Assuming that the rifts are the result of a steady state process, and that the area of the rifts increases linearly with time, and that the speed of the ice shelf is approximately uniform, the area of the each rift increases by

$$\frac{20\,km\,long \times 2\,km\,wide \times 1/2 - 10\,km\,long \times 0.5\,km\,wide \times 1/2}{60\,km \div 600\,m\,yr^{-1}} = 17.5\,km^2/100\,yr$$

If the thickness of the ice is 800 m, and all four rifts are increasing in area at the same rate, then this leads to an ice-balance error +0.56 km ^{3}/yr: Assuming steady state and ignoring the rifts would lead you to estimate that the ice shelf was gaining this amount of ice. By comparison, the total ice-front discharge contributes around -12 km ^{3}/yr to the ice shelf mass balance.