Effect of Rheology on the Thinning Function

The thickness of an annual layer in an ice core record is the result of the accumulation rate at the time of deposition and the dynamic thinning due to ice flow (affected by rheological properties of ice) and overall thinning or thickening of the ice sheet. Often we wish to infer the past accumulation rate from the layer thickness; to do this we much determine the *thinning function*, $\lambda(a)$, for a particle of ice of age a.

To get the age-depth relationship for an ice core site, we integrate the vertical velocity along the flow path of the ice particle.

$$A(x, z, t_o) = \int_h^z \frac{1}{w(x'(t'), z'(t'), t')} dz' \tag{1}$$

The instantaneous thinning of an annual layer of thickness *lambda* is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \dot{\epsilon}_{zz}\lambda \tag{2}$$

$$\dot{\epsilon}_{zz} = \frac{\mathrm{d}w}{\mathrm{d}z} \tag{3}$$

Since the strain rate varies with time and depth as the layer moves, we integrate to get the final thickness of the layer. Ice of age A is

$$\lambda(A) = \dot{b}(A)\exp[\int_{-a}^{0} \dot{\epsilon}_{zz} dt] \tag{4}$$

The mathematically simplest age-depth scale is the Nye timescale, which assumes that the strain rate is constant with depth and, therefore, the vertical velocity is zero at the bed and decreases to $-b$ at the surface):

$$\dot{\epsilon}_{zz} = \frac{\mathrm{d}w}{\mathrm{d}z} = \frac{-\dot{b}}{H} \tag{5}$$

Given a uniform accumulation rate through history, the layer thickness for this profile of strain rate is

$$\lambda(A) = \dot{b}(A)\exp[\frac{-b}{H}A] \tag{6}$$

The Nye timescale may be most appropriate for an ice divide where the bed is sliding. These plots show the related curves for the "Nye" Timescale along with two other strain rate profiles more appropriate for a frozen bed. The dashed line is the Dansgaard Johnsen profile and the dotted curve is for a divide with strong anisotropy in the deeper layers.

Questions:

1. Draw your best estimate of the vertical velocity profile, the age depth profile, and the layer thickness with age relation for the other two vertical strain rate profiles. (General shape and position of curves relative to the Nye curves).

The mathematical answer to this just involves some integrations and thinking them through qualitatively - a good skill in itself! Most ice cores are drilled near ice divides, and ice divides provide the context for the thought experiment. The vertical strain rate profile in an ice sheet typically has the fastest thinning in the upper layers, where the layers are being "stretched" horizontally by the flow. To satisfy continuity, the vertical strain rate must equal the divergence of the horizontal velocity (i.e. what you stretch horizontally must match your thinning vertically). At an ice divide, this means that the layers are pulled one way off one side of the divide and the other way off the other side, so at the divide they are being stretched. This is highest in the upper layers because that is where the fastest (diverging) velocities exist. The vertical strain rate in deeper layers must stay the same or get smaller at the bed.

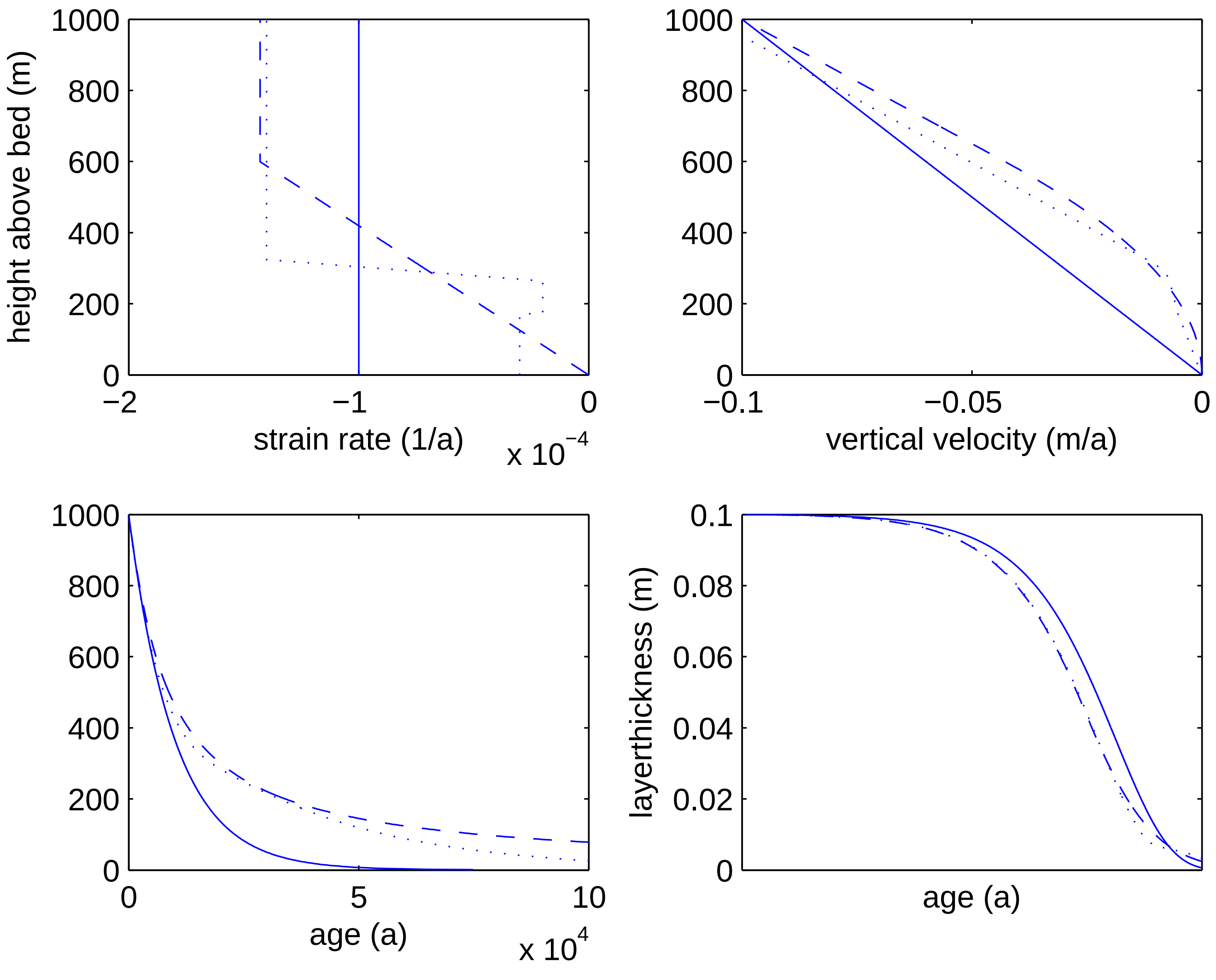
The vertical strain rate is the derivative (slope) of the vertical velocity profile. The highest vertical velocities are near the top, and they decrease towards the bed. The vertical velocity must be zero at the bed unless you have substantial subglacial melting. For ice with a "stiff" layer deeper down, this stiffness will shift more deformation higher in the ice column, so the curvature of the velocity profile will be higher.

Similarly, if it is difficult to thin the stiff deep (old) layers, the ice sheet will "store" a substantial amount of the old ice around, so the age depth scale will be altered accordingly.

Finally, with stiff ice that is difficult to thin you will end up with thicker layers at depth than if the ice were softer.

2. The strain rate profile with strong anisotropy (dotted), is not a profile which will remain steady through time. In reality, the low strain rate regions will advect with the ice as it moves deeper (the low strain rate regions are roughly coincident with ice-age ice. How will a changing strain rate profile affect your conclusion above about the layer thicknesses?

The way we solved this above is assuming that the profile of viscosity remained stationary. A better interpretation is that the stiffness is a property attached to the layer, regardless of the depth of that layer, then the stiffer layers will always be thicker for the same accumulation rate, no matter how deep they are. The ice above and below the stiff layer may deform (and thin) substantially. This will make the interpretation of a depth-age scale in terms of accumulation history quite challenging. In theory, this could also cause a "boudinage" effect in ice sheets (it has never been observed).


height above bed (m)
1000
800
600
400
200
0
−2
−1
0
strain rate (1/a)
x 10^{-4}
1000
800
600
400
200
0
−0.1
−0.05
0
vertical velocity (m/a)
1000
800
600
400
200
0
0
5
10
age (a)
x 10^{4}
layerthickness (m)
0.1
0.08
0.06
0.04
0.02
0
age (a)