MOTION AND CALVING AT LECONTE GLACIER, ALASKA

A

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MOTION AND CALVING AT LEOCONTE GLACIER, ALASKA

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Abstract

An analysis of motion and calving in the terminus region of LeConte Glacier delineates controls which are important to tidewater glacier stability. Ice velocities in this region are quite high; at the terminus they exceed 27 m d$^{-1}$. Our analysis reveals fluctuations in velocity that are forced by ocean tides, surface melt and precipitation. However, the overall velocity is steady over seasonal time intervals. LeConte’s terminus position varied substantially, even given this steady ice influx, establishing a correlation between the calving flux and the terminus position (flux out). Although this correlation is largely numerical, the occurrence of calving events is not purely stochastic. Calving occurs as floatation is approached, and multiple short-lived triggers may force calving events by promoting a buoyancy instability. These triggers may include the tide, water input, and water depth. Flexure of the nearly floating portion of the glacier promotes crevasse growth, and helps to initiate calving.
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Preface

This thesis is a result of two periods of field work during 1999 at the terminus of LeConte Glacier, in Southeast Alaska. For 32 days in spring and 5 days in fall, we monitored ice motion and calving in efforts to study the temporal variability of these two processes. Chapter 2 describes the velocity study, while Chapter 3 focuses on calving dynamics, with references to Chapter 2 as necessary. This thesis is prepared for submission of two stand-alone papers in *Journal of Glaciology*; therefore some overlap is necessary between chapters. Throughout the text, ‘we’ is used to refer to the three authors, myself, Keith Echelmeyer, and Roman Motyka. This project was supported by NSF grant OPP 9877057.

Many people deserve thanks for helping me complete this thesis. At the top of the list is my advisor, Keith Echelmeyer, who went out on a limb and accepted me as a graduate student, even though I had no idea what $\nabla$ meant when I arrived. Since then, he has taught me much more than $\nabla$ and we’ve had many thought provoking discussions always swinging wildly from iceberg calving equations to the most recent climb one of us had completed.

Roman Motyka was always dragging me back to reality, making me think of physical meaning instead of equations. He was also key in initiating a wonderful side project on Mendenhall Glacier, one that never ceased to be exciting and beautiful. I enjoyed my times in Southeast with Roman, where the rain is just part of the joy. Will Harrison always had wonderful ideas, and was always making sure I had considered the uncertainty, which I never had. Doug Christensen was always convincing me to sign up for classes that focused on something other than ice.

Great people helped with field work at LeConte. Paul Bowen for initiated the research at LeConte. Joel Johnston was a super dedicated intern and enthusiastically digitized hundreds of photos, with great results. During the field work, it wasn’t always easy to go out at 1 a.m.,
into the maelstrom for 30 minutes of surveying. Bryan Hitchcock, Shannon Siefert, Patty DelVecchio and others were always ready to help, especially if we could continue our ski trips between surveys. Bryan gets extra thanks for always being ready for some ridiculous boondoggle attempt at random peaks, we had some good ones! Wally, the pilot was incredible. He claims I took several years off his life placing markers on ice pinnacles, but without his steady had, these papers would be pretty short!

Martin Truffer and Dan Elsberg certainly got tired of my personally formulated rules of algebra, but were always ready to indulge in some wild conversation pertaining to the latest idea that popped into our heads, or to plan some random adventure. Not to mention all the fun times we had slogging around glaciers and peaks across Alaska. Aaron Pearson and Dan McNamara were always ready to jump in the truck and head to the Delta’s, where the scenery was often better than the climbing, but it always beat spending the weekends in town.

Last, thanks to Alaska’s mountains and glaciers for employing me for the last three years, and providing endless opportunity to get scared clinging to some face, pillar, or slab, satisfying the hunger, and providing the necessary focus to work again, until the next climb.
Chapter 1. Introduction

Background
The concept of a tidewater glacier cycle is well established (Post, 1975), and there are many observations of tidewater glaciers that are either slowly advancing or rapidly retreating (e.g. Mercer, 1961; Meier and Post, 1987; Alley, 1991; Kamb and others, 1994; Meier and others, 1994; Post and Motyka, 1995; Warren and others, 1995). Although considerations of the asynchronous behavior of these glaciers and their apparent independence from climate forcing are numerous (Mann 1986; Porter, 1989; Post and Motyka, 1995; Motyka and Beget, 1996), processes that govern tidewater glacier stability, the initiation of retreat and the associated changes in velocity remain poorly understood. Specifically, the prediction of the rate of calving, which is closely related to glacier stability, as a function of some measurable parameter such as water depth, effective pressure at the bed, ice velocity or runoff, is not yet feasible (Meier, 1997).

The rapid retreat is accomplished by an increased rate of calving. Such retreat has the potential to drastically alter the volume of glaciers and ice sheets (Heinrich, 1988; Broecker, 1994) and force a potentially large rise in global sea level (Meier, 1984; 1990). Much of what is known about the retreat phase of tidewater glaciers stems from research at Columbia Glacier, which is a grounded, temperate tidewater glacier in southcentral Alaska. In 1975, Post predicted a rapid retreat of this glacier would result if the glacier backed off its submarine shoal. The retreat was initiated, and since 1983, the terminus has retreated over 10 km (Krimmel, 1997). Meier and Post (1987) surmised that this retreat may have been initiated by calving, followed by a reduction in backstress and an increase in velocity. Contrarily, van der Veen (1996) used the same data set to suggest that rapid flow, rather than calving, initiated the retreat by promoting thinning and an increase in glacier buoyancy.
In addition to the debate centered on the initiation of the retreat phase, the forces that promote calving remain elusive. Using observations from 15 Alaskan glaciers, Brown and others (1982) postulated a relationship between water depth at the terminus and calving rate. This hypothesis has been generally quoted, but observational evidence suggests it does not universally apply. For example, Sikionia (1982) noted that the relation breaks down at short time scales, and is replaced by a relationship with runoff. Van der Veen (1996) questioned the use of this relation for long time periods; he argued that Columbia Glacier has retreated into shallower water during times of increased calving, and suggested that the water depth hypothesis may only hold for glaciers near steady state.

Another possible calving relation, first suggested by Meier and Post (1987), suggests that the calving rate is related to the effective pressure. In this scenario, the terminus retreats to a point where the effective pressure at the bed is near zero. Van der Veen (1996, 1997) has further developed this idea, incorporating water depth, by suggesting that a glacier’s height above buoyancy may be the parameter controlling glacier stability and calving rate, where the glacier retreats to some height above buoyancy that is fixed (~50 m).

After the initiation of the calving retreat at Columbia glacier, the ice velocity began to increase markedly. Detailed surveys of ice motion in the terminus region documented the trends and variations in motion over multiple time scales, then investigated the role of ice motion in promoting calving and retreat. Meier and Post (1987) and Krimmel and Vaughn (1987) describe the long-term speed up as the retreat began, as well as seasonal fluctuations in velocity and terminus position. They show that velocity and glacier length vary seasonally, such that maximum length is nearly concurrent with minimum velocity. Walters and Dunlap (1987) and Walters (1989) describe short time scale variations in velocity, and relate these to changes in tidal stage and meltwater inputs. More recent field observations suggest that
variations in rapid motion may be controlled by water storage at the bed (Fahnestock, 1991; Kamb and others, 1994, Meier and others, 1994).

Observational studies on other grounded tidewater glaciers are limited, but they do offer valuable comparative information. Warren and others (1995) have estimated calving fluxes at Glacier San Rafael, Chile, and used these estimates to relate calving to measured parameters, including the tide, wave action and water chemistry. Although they found no simple relationships, their calculations do suggest that submarine melting may be important. Recently, analytical and numerical modeling experiments have investigated both the restraining forces of the submarine shoal (Fischer and Powell, 1998) and the roll of water depth in calving (Hanson and Hooke, in press). Hanson and Hooke argue that deep water may facilitate an oversteepened ice cliff, resulting in rapid calving, but conclude that calving is likely governed by multiple forcings. Fischer and Powell have shown the importance of the restraint provided by terminal moraines; their model suggests that they provide the dominant restraining force when the moraine height reaches 20-30% of the local maximum water depth.

In this thesis, we discuss detailed observations of ice motion and calving at LeConte Glacier, which is a rapidly retreating, grounded tidewater glacier located in southeast Alaska. The glacier is approximately half the size of Columbia Glacier, and is located in a similar maritime climate. We seek to define relationships between ice motion and calving, as well as formulate cause-effect relations between parameters such as tidal stage, precipitation, ice ablation, and changes in both velocity and calving. By performing measurements over short time scales, we attempt to identify mechanisms which may explain calving at both short and long time scales, a task which thus far has not been accomplished. Our studies, when compared with those on other tidewater glaciers, allow differentiation between local and global processes that control tidewater glacier dynamics.
Setting

LeConte Glacier is located approximately 35 km east of Petersburg, Alaska (Fig. 1a), and is the Northern Hemisphere’s southernmost tidewater glacier. In 1994, after a 32 year period of stable terminus position, LeConte Glacier began a rapid retreat. Since then, the glacier has retreated about 2 km (Motyka, personal communication). Drastic thinning has accompanied the retreat, averaging 2.4 m a\(^{-1}\) over the entire glacier, with a total thinning in the terminal region of ~250 m over the last 40 years (Echelmeyer and Harrison, unpublished data, 1999). The retreat was first noticed by P. Bowen of Petersburg High School. His students have surveyed the position of the glacier terminus on an annual basis since 1983 (Bowen, personal communication, 1999). These surveys, together with photogrammetric analyses, surveyed terminus positions and ice velocities (Motyka and others, in preparation, 2000), document long-term trends in velocity and terminus position.

LeConte Glacier mantles the Coast Range Batholith, a complex of resilient granodiorite mountains. The glacier is approximately 35 km long, covers an area of 469 km\(^2\), and has a large accumulation area ratio (nearly 0.90, Post and Motyka, 1995). Ice flows from a large accumulation area on the Stikine Icefield (accumulation area extends from an elevation of 2600 to 920 m) into a deep, narrow, fjord. Bathymetric data acquired about 200 m from the terminus (Motyka and Hunter, unpublished data) show that the steep-walled fjord has a maximum depth of ~270 m below sea level. The glacier centerline is shifted approximately 150 m south of the deepest part of the channel (Fig. 2a). The terminus is completely grounded, with the majority of the terminal ice lying below sea level. A comparison of surface elevations from 1996 and 2000 (airborne altimetry data, Echelmeyer, unpublished data) shows that at the present location of the terminus the glacier has thinned by approximately 125 m since 1996; during the same period, the glacier retreated about 1 km. This thinning lead to an 85%
reduction in the height above buoyancy at this location. Currently, the glacier terminus is only 25 m in excess of floatation.

The near-terminus surface topography is steep, with surface slopes ranging from 8° to 10°. Heavy crevassing dominates the lower 8 km of the glacier, with the last 4 km composed mainly of seracs and ice pinnacles in a rapidly changing configuration. The terminal ice cliff has an average height of 60 m above the sea surface (Fig. 2a). A terminal moraine exists about 2 km down fjord of the present terminus, marking the most recent (1962-1994) position of terminus stability. The water depth at the moraine shallows to about 160 m. Given the high erosive strength of the surrounding bedrock, formation of such a submarine moraine is likely a much slower process than is typical for many other tidewater glaciers in Alaska, which generally erode soft sedimentary or metamorphic rocks.

Surface velocities near the terminus have been steadily increasing since research began. They currently exceed 27 m d⁻¹. The velocity is much lower 7 km upstream, where the centerline velocity is approximately 3.5 m d⁻¹. Thus, this lower region of the glacier is subject to extreme longitudinal strain rates, at some locations they exceed 5 a⁻¹, and are responsible for the heavy crevassing in this region.
Chapter 2.

Short-term flow dynamics of a retreating tidewater glacier\(^1\)

Introduction

During the spring and the fall of 1999, we established a field camp above the terminus of LeConte Glacier (labeled LAKE in Fig. 1b). From this camp, we measured ice motion in the terminus area, while simultaneously monitoring the terminus position and iceberg calving. Intervals between surveys were short, enabling analyses at several time scales ranging from semi-diurnal and diurnal to lower ($\leq$ 1 cycle/week) frequencies. In addition, we measured tidal stage, ablation, air temperature, and the bathymetry of the fjord. Qualitative observations of subglacial discharge were made, and precipitation data were obtained from Petersburg and supplemented with measurements made at the glacier. This chapter first discusses general flow patterns, followed by analyses over semi-diurnal to seasonal time scales. Ice velocity in the terminus region exhibits response to multiple short time scale forcings, ranging from semi-diurnal tides to isolated precipitation events. Also present, but sometimes masked by stronger tidal variations, are diurnal variations in motion driven by meltwater input. Separation of these various forcings is accomplished via signal filtering and harmonic analysis of ice motion. We then attempt to identify the origin of these velocity variations.

Observations and methods

Motion

Horizontal and vertical ice motion was monitored for 37 days (May 2 through June 4; August 26-30) at several markers. We used optical survey methods, employing a 1 s theodolite and a long-range electronic distance meter. Tetrahedral markers, about 1.5 m tall and equipped

\(^1\) Prepared for submission in *Journal of Glaciology*
with reflecting prisms and darkness-activated flashing beacons, were placed on seracs using a helicopter. Over the course of the study, we deployed a total of eighteen markers from 0 to 7 km from the terminus. Thirteen of these markers were placed near the longitudinal centerline of the glacier (Fig. 1b), and the remaining five were set on a transverse profile across the width of the terminus (Fig. 2b). Because the five transverse markers were not equipped with reflecting prisms, the distance to these markers was only measured one time, when they were placed on the glacier. They were then surveyed for only two days to obtain a transverse profile of surface velocity (Fig. 3). Due to serac instability and calving losses, some markers were periodically reset; new positions were chosen as close to the initial marker positions as possible in order to investigate the temporal changes in motion at a given point in space (an Eulerian reference frame). We labeled the centerline markers A through G (replacement markers are labeled with an asterisk, e.g. B*), plus Bend and Gate (Fig. 1b). T1-T5 were the transverse markers (Fig. 2b). A longitudinal coordinate system $\xi \in [0, 9 \text{ km}]$ was defined with the origin ($\xi = 0$) located just upglacier from Gate, where the glacier enters a well-defined constriction (note that $\xi = 0$ is near the average 1990’s equilibrium line). $\xi$ is positive towards the terminus (Fig. 1b); $\xi = 7$ marks the position of the May 1999 terminus, and $\xi = 9$ the 1962-94 position.

As we were interested in identifying any tidal forcing of glacier speed, we attempted motion surveys at few hour intervals in order to satisfy the Nyquist sampling criteria, which states that the sampling interval should be less than or equal to $1/[2^* \text{ folding frequency}]$. For a semi-diurnal cycle, this requires that sampling be performed at least 4 times per day (Godin, 1972). When possible, we surveyed at two to three hour intervals, with a three to six hour gap at night. Therefore, our surveys generally satisfy the sampling criteria, but the sampling interval was not constant and there were data gaps.
The weather during May, 1999, was unseasonably poor, and markers sometimes became obscured by clouds or fog during a survey. At times, heavy rain, snow and wind also made it difficult or unreasonable to survey. However, the resulting time series of motion are relatively complete, as can be seen in Figures 4a and 5a.

Distance measurements were corrected for changes in air temperature and pressure during each survey. Under best conditions, angle measurements were accurate to two seconds of arc, distances to ± 3 to 5 mm, and times of surveys to ± 5 seconds. Given these measurement errors, the estimated errors in the surveyed positions range from ± 3 cm in good conditions to ± 6 cm in poor surveying conditions. Over 3 hr time intervals these correspond to errors in velocity of \( \pm 0.34 \text{ m d}^{-1} \) and \( \pm 0.68 \text{ m d}^{-1} \), respectively. Additional errors were occasionally introduced by marker tilt or rotation on the small serac tops. The average error in vertical position is estimated to be ± 5 cm.

Further upstream at Bend and Gate (Fig. 1b), we deployed dual-frequency GPS receivers, which collected position data six and two times daily, respectively, for the duration of the study. A third receiver was deployed at a fixed benchmark near LAKE, allowing post-processing of the data to a positional accuracy of ± 3-5 cm. Rotation of the antennas as crevasses opened near the markers may have caused some degradation of this accuracy. A few gaps exist in this otherwise continuous record because of heavy snowfall and subsequent power losses.

Tide

Complete knowledge of the ocean tide at the glacier terminus is critical to our analyses of velocity and calving. The closest continuously operating tide gauge to LeConte Glacier is located in Ketchikan, Alaska, over 100 km from the glacier. This long baseline results in amplitude and phase differences between the Ketchikan and LeConte Bay tides. To more
accurately determine the tide in LeConte Bay we obtained NOAA water level data gathered in LeConte Bay during the spring of 1997 (http://www.co-ops.nos.noaa.gov/data_res.html). We also installed our own tide gauge at the NOAA location during August 1999. Using these data, we were able to define the local tide at any time during our study, as described in detail later. These results (Fig. 4b) show that the tide in LeConte Bay has a strong semi-diurnal component, with two highs and lows of unequal magnitude each day. Peak-to-peak amplitude varies from about 2.5 to 6 meters. In what follows we reference the ‘tidal amplitude’, which we take to be the range between the average of the two high tides and the average of the two low tides each day.

Other observations

Hourly air temperature and ice ablation were measured using a sonic ranger on a tributary glacier about 3 km from the terminus and 530 m above sea level. Temperatures were accurate to ±0.4 °C, while ice ablation was accurate to ±1 cm. The ablation rate (time derivative of the ablation data; Fig. 4c) exhibits clear diurnal variations, even though the extremely variable weather and long duration rain events during the study interval introduce large variability in the timing (±0.2 d) and magnitude of the peak ablation rate. Thus the ablation rate has a broad spectral peak, centered around 1 cycle d⁻¹. Negative values represent snowfall events.

Daily precipitation was measured by the National Weather Service at Petersburg Airport, about 35 km from the glacier. During the study interval, the largest precipitation events occurred on days 144, 140-41, and 151. We also measured precipitation for nine days (J.D. 145-154) at the glacier terminus. The two records generally follow similar trends, but the magnitude of the precipitation at the glacier was often twice that measured in Petersburg (Fig.
However, at times the precipitation patterns may differ markedly; the rain event recorded in Petersburg on day 151 arrived at the terminus one day earlier.

Water discharge from a tidewater glacier is difficult to monitor, but it plays an important role in basal hydrology and glacier motion. To address this issue, we made qualitative estimates of upwelling at the terminus. We observed the timing and magnitude of silt-laden freshwater plumes in the fjord, just downstream of the terminus as a proxy for discharge (Fig. 4e). The presence of these plumes was also recorded in the time-lapse images that were used to study calving activity. Upwelling plumes were easily distinguishable as they would drive ice bergs and ‘brash’ ice away from the terminus. A strong upwelling event would create whitecaps in the terminus fore-bay. A lack of an upwelling left the terminus region packed with ice (Fig. 6). We ranked the magnitude of upwelling on a qualitative scale where a full plume is represented by magnitude 5, and the absence of upwelling represented by a zero.

**Features of the motion**

Velocity data (with no smoothing) are presented in Figure 4a. Several noteworthy features deserve attention. First, the velocities of all markers are quite large, ranging from ~10 m d\(^{-1}\) at \(\xi=4\) km to over 27 m d\(^{-1}\) at the terminus. Second, a large longitudinal velocity gradient is present. We attribute this gradient to thickness gradients as ice flows to the terminus, as well as a substantial reduction in glacier width as the terminus is constrained by the valley walls. Third, semi-diurnal variations in surface velocity, with amplitudes up to 5% of the mean, are clearly visible for markers A/A* (where A/A* is the combined record for markers A and A*), B* and D. Non-tidal diurnal variations in velocity with amplitudes up to 0.5 m d\(^{-1}\) (5-8% of mean) are visible upstream from the terminus, especially at Bend and Gate. Finally, a low frequency variation, centered around day 145 and lasting ~3 days, is present in all velocity records. This event follows a period of heavy rain.
To develop a basis of glacier flow in the terminus region, we have calculated the basal shear stress, $\tau_b$. Because large longitudinal stress gradients are present here, the calculation strongly depends on the length over which values of ice thickness and surface slope are averaged. However, the best averaging lengths to perform these calculations over is unclear. For this reason, we performed the calculation over variable averaging lengths, ranging from 0.5 to 2.5 km. We used the known bathymetry, the effective cliff height, and assumed a horizontal bed to arrive at an average thickness of 375 to 475 m. The surface slope average varies between 8° and 10°, and the appropriate shape factor for this for this steep walled parabolic channel is 0.53. The calculated basal shear stress, ranges from $2.1 \times 10^5$ to $2.8 \times 10^5$ Pa. If we choose a longitudinal coupling length of 0.5 km (for reasons discussed later), the calculated basal shear stress is $2.5 \times 10^5$ Pa. This gives a surface velocity due to internal deformation of 2.1 m d$^{-1}$ (Fig. 7). As this is only 8 to 20% of the surface velocity in the lower 3 km of the glacier, we conclude that basal motion dominates the ice flow. Direct observations of basal sliding, where a small tongue of ice flows around a rock cleaver near the terminus, demonstrate that marginal sliding is present and possibly even dominant.

**Flow around a bend**

As ice approaches the terminus of LeConte Glacier, it flows around a sharp bend (the centerline radius of curvature is about 1.1 km, while the glacier width is only 1 km), causing the flow direction to change by more than 90° (Fig. 1b). The transverse velocity profile near the end of this bend, shows that the flow maximum is shifted outward by about 80 m from the glacier centerline (Fig. 3). Theory describing ice flow in a curving channel (Echelmeyer and Kamb, 1987) accurately predicts this outward shift. However, the observed shape of the velocity profile does not match the theoretical prediction because the flow is dominated by sliding in this case. According to their theory, the stress centerline should be shifted toward the
inner margin, and the surface slope should vary across the glacier, with maximum slope on the inside of the bend. Such a variation in slope is also observed.

**Strain rate**

As ice approaches the terminus, it is subject to large longitudinal gradients in velocity and strain rate (up to $5 \text{ a}^{-1}$; Fig. 8). These strain rates are extremely large, being about an order of magnitude greater than those observed at Columbia Glacier (Venteris and others, 1997). The strain rate reaches a maximum value of $6 \text{ a}^{-1}$ about 200 m upglacier from the terminus; it then drops rapidly by approximately an order of magnitude as the terminus is approached. This maximum occurs less than one centerline ice thickness (centerline thickness $\sim 320$ m) back from the terminus, at a distance which is approximately equal to the average ice thickness ($\sim 220$ m) at the terminus. These high strain rates cause heavy crevassing and thinning in the terminus region. The recent thickness change at the terminus is known from repeat airborne profiles in 1996 and 2000 (Echelmeyer, unpublished). Using this, we estimate the thinning caused by longitudinal stretching alone. With a measured time-averaged thinning rate in the terminus region of $30 \text{ m a}^{-1}$, an ablation rate of $9-11 \text{ m a}^{-1}$, and given the maximum estimate of bottom melting at $5.5 \text{ m a}^{-1}$ (discussed later), the thinning rate caused solely by longitudinal stretching is $19 \text{ m a}^{-1}$, or about 60% the measured thinning rate.

**Seasonal variations in speed**

**Table 1** gives a comparison of velocity measurements made at similar locations on the glacier surface at different times. These comparisons were made between markers that were located less than 2 m apart along the flow direction, with the distances in the last column of **Table 1** being the separation of the markers used for the two epochs, measured transverse to the direction of flow. Marker A (first row in **Table 1**) shows no change in speed from pre-melt conditions (and no liquid water at the surface) into the early melt season. The other comparisons span a three month interval (May to August), which is nearly the entire melt
season. The early part of this interval was characterized by precipitation as snow and little melt on the glacier; at the end of the interval, new snow was accumulating on the glacier as low as Gate (700 m HAE). Over this interval, the glacier speed at these three locations was nearly constant; any differences are likely accounted for by the transverse position of the markers (especially marker E). Additional data comes from the continuous GPS record obtained at Gate, which shows short-term variations, but no seasonal change in speed over the same three month interval (Fig. 9). Thus, we believe that there are no substantial seasonal variations in speed over the lower 7 km of the glacier.

**Table 1.** Seasonal changes in speed. The velocities for markers moving along similar flowpaths at different times are shown. Marker separation distances are transverse to flow.

<table>
<thead>
<tr>
<th>Marker</th>
<th>Average initial time (d)</th>
<th>Average initial speed (m d(^{-1}))</th>
<th>Average final time (d)</th>
<th>Average final speed (m d(^{-1}))</th>
<th>Time interval (d)</th>
<th>Marker separation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (7 km)</td>
<td>May 10</td>
<td>26.7</td>
<td>May 17</td>
<td>26.8</td>
<td>7</td>
<td>25</td>
</tr>
<tr>
<td>B (6.7 km)</td>
<td>May 9</td>
<td>25.8</td>
<td>Aug. 29</td>
<td>25.5</td>
<td>112</td>
<td>50</td>
</tr>
<tr>
<td>E (5.5 km)</td>
<td>June 2</td>
<td>10.7</td>
<td>Aug. 28</td>
<td>13.7</td>
<td>86.5</td>
<td>445</td>
</tr>
<tr>
<td>G (4.3 km)</td>
<td>May 27</td>
<td>10.6</td>
<td>Aug. 29</td>
<td>10.7</td>
<td>94</td>
<td>145</td>
</tr>
</tbody>
</table>

**Short-term fluctuations in motion**

**Harmonic analysis of the tide**

A standard technique for tidal analysis, often referred to as ‘harmonic analysis’ (Godin, 1972; Foreman, 1977), was used to analyze the local tide and, subsequently, the ice speed data. In this analysis we assume that a time series can be partially represented by a sum of discrete sinusoids, each with a prescribed frequency (as governed by tidal forces), \(\omega_i\) (rad h\(^{-1}\)), but with unknown amplitude, \(A_i\), and phase, \(\phi_i\):

\[
\mathcal{H}(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t - \phi_i) + M + \text{noise}
\]  

(1)
where $H(t)$ is the tide (or, later, the ice speed, or calving flux), $t$ the time in days, $M$ the mean tide (or, later, speed, flux) and the subscript $i$ ranges over the $N$ constituents assumed to make up the time series. Harmonic analysis then proceeds by non-linear least squares, to solve for the unknown amplitude and phase of each constituent, with its prescribed frequency. The computer code that we used also determines the reduction of variance (ROV) in a stepwise fashion as each constituent is added to the analysis. Through this process the relative strength of each individual constituent can be resolved. The statistical significance of the predicted time series was determined using the reduced chi-squared test (Bevington, 1969). A residual time series (equal to the input series minus the $N$-component predicted series) was also calculated.

The primary tidal constituents are either semi-diurnal or diurnal. Notation for these constituents consists of a letter representing the source (lunar, solar), followed by a subscript delineating the approximate frequency (diurnal = 1, semi-diurnal = 2) (Godin, 1972). For example, $M_2$ is the principal lunar semi-diurnal constituent.

We used the 1997 NOAA tide stage observations made in LeConte Bay to solve for the amplitude and phase of the dominant tidal constituents (Fig. 10a). This solution was then checked by using these constituents (and their determined amplitudes and phases) to predict, via Equation (1), the tide during the time period when we measured the tide in 1999. We also compared the predicted tide with the NOAA estimate for Petersburg (Fig. 10b). The fit to both sets of observations was excellent. Amplitude and phase discrepancies exist with respect to Petersburg; this is because the tidal estimate for Petersburg is derived from observations made in Ketchikan.

Our analysis shows that, in order of decreasing importance, the six strongest constituents of the LeConte Bay tide are $M_2$, $K_1$, $S_2$, $L_2$, $N_2$ and $O_1$ (Table 2). These constituents determined $\sim98\%$ of the variance in the tide signal. The semi-diurnal constituent
M₂ dominates the tide (81% ROV), followed by the lunar diurnal constituent, K₁. The tide is relatively free of complicated shallow water, overtide constituents at higher frequencies.

*Table 2. LeConte Bay tide. Tidal constituents, their periods and strength in the local tide.*

<table>
<thead>
<tr>
<th>Tidal Constituent</th>
<th>Period (hr)</th>
<th>Variance Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₂</td>
<td>12.421</td>
<td>81%</td>
</tr>
<tr>
<td>K₁</td>
<td>23.934</td>
<td>6.5%</td>
</tr>
<tr>
<td>S₃</td>
<td>12.000</td>
<td>6%</td>
</tr>
<tr>
<td>L₂</td>
<td>12.192</td>
<td>2%</td>
</tr>
<tr>
<td>N₂</td>
<td>12.658</td>
<td>2%</td>
</tr>
<tr>
<td>O₁</td>
<td>25.819</td>
<td>1%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>98%</td>
</tr>
</tbody>
</table>

Given the amplitude and phase for these six constituents, we can predict the tide at any time using *Equation (1)* to an estimated accuracy of 0.25 m, with little or no phase discrepancies, except possibly during times of extreme high or low atmospheric pressure. In *Figure 4b* we show this predicted tide for the study interval.

**Short-term variations in horizontal motion**

In this section we describe the methods and results of our analyses of horizontal speed, *U(t)*, over tidal to several day time periods.

**Signal filtering**

Prior to the analysis, we smoothed and filtered the speed series that are shown in *Figure 4a*. An Eulerian reference frame was approximated by removing the effects of the large longitudinal velocity gradients shown in *Figure 8*. Next, we fit cubic splines to the data and sampled them at three hour intervals, which was approximately equal to our nominal surveying interval. These series were then subjected to a low pass filter with a cutoff period of twenty-four hours to isolate the portion of the signal below tidal frequencies. The filter used was

\[ A_n^2 A_{n+1} \],

where

\[ A_n = \frac{1}{n} \sum_{j=0}^{n-1} U(t + j\Delta t) \]  

(2)
(Godin 1972, p.65). $U$ is the ice speed, $\Delta t$ our sampling interval (3 hr), and $n = 8$. This filter is robust and does not suffer from aliasing (Walters and Dunlap, 1987). Now isolated, the low frequency part of the signal was subtracted from the splined interpolant, leaving the high frequency portion of the signal, $U_{\text{high freq}}$.

\[
U_{\text{high freq}} = U(t) - A_s^2 A_s[U] 
\]  

(3)

We describe these two parts of the signal in turn.

**Low frequency variations**

The low frequency velocity time series, $A_s^2 A_s[U]$, for each marker are shown in Figure 11. The most prominent feature of these series is the speed-up event centered around day 144.5, which lasted about three days and had an amplitude of 5% to 13% of the mean speed at a marker. The onset, duration, and time of peak speed are similar for each of the markers, with a variance of less than 0.5 d in their timing.

A correlation (correlation coefficient, $C = 0.54$ to 0.66 with a phase lag of +1.0 d) exists between the low frequency speeds and excessive water input provided by heavy precipitation, such that precipitation events precede the maximum speed. After some speed-up events, a few of the marker speeds decrease to a level which is lower than before the event (i.e. marker B*, D, G). These events are similar to the “extra slowdowns” described by Meier and others (1994) at Columbia Glacier, and imply that the direct correlation between water input and speed is not causal. If it were, events such as “extra slowdowns” would not exist, since water input levels (hence speeds) are likely return to initial levels after a storm, following the reasoning of Walters and Dunlap (1987) and Kamb and others (1994). The magnitude of a given speed-up does not appear to be strictly governed by the magnitude of water input, but speed-ups do appear to occur more frequently near the terminus where the cumulative basal
water flux is maximized. After the periods of elevated water input, decay of the speed-up occurs as water is discharged from the glacier.

To determine the possible effects of water storage, we estimate a crude water storage index (Fig. 12). This index was compiled by differencing water inputs and outputs each day. Inputs and output were included in the index only when they were above base levels. For water inputs, this includes any recorded precipitation and any surface melting in excess of the lowest daily melt rate maximum. Any visible upwelling was considered as an above base level discharge. The speed-ups appear to be centered on the peak in the water storage index, as was also found by Fahnestock (1991) and Kamb and others (1994).

Semi-diurnal variations
We applied harmonic analysis to the high frequency time series, \( U_{\text{highfreq}} \), following Equation (1). These results are presented in Table 3, where the total reduction of variance (ROV), and the ROV from the semi-diurnal M\(_2\) constituent alone are given, as well as the amplitude and phase for each constituent for each marker. Markers A/A* and B* demonstrate the best overall ROV; and there is an upglacier decay in the overall ROV. Figure 13 shows the input series and predicted series for each marker. In some cases, the predicted series appears to match the observations quite well (\( \chi^2 \) for markers A/A* and B* yields <1% probability of a random fit). However, even in these cases, the ROV is less than 50%, indicating that the velocity fluctuations are not completely tidal in nature. The upstream markers have a much higher probability of a random fit (~50%). Marker C has an abnormally noisy record and a poor solution.

Table 3. Harmonic analysis of horizontal motion. Amplitude (A) and phase (\( \phi \)) relations for each marker and the tide using the six strongest tidal constituents. The total ROV and the M\(_2\) ROV are given below each label in parentheses.
The results in Table 3 show that, as in the case of the tide, a majority of the fluctuations in speed are semi-diurnal ($M_2$) in nature. We are unable to resolve the other constituents, as their signal is either too weak or contaminated by other forcings (also noted by Walters and Dunlap, 1987 on Columbia Glacier). Because the $M_2$ signal is the clearest, and is not contaminated by other forcings (such as diurnal melt), we take the $M_2$ response in Table 3 to represent the tidal effect on velocity at each marker.

The $M_2$ phase angle for the tide is $16^\circ$, and the average $M_2$ phase angle for markers A/A* and B* is about $-160^\circ$. Thus, the phase difference between the two is $176^\circ$, which corresponds to a peak tide/low speed relationship, with virtually no phase lag. We also examined the tide-speed phase relationship by cross correlating the two time series. These results show the ice speed is within one hour from being $180^\circ$ out of phase with the tide (Fig. 14), in good agreement with the $M_2$ harmonic analysis.

The amplitudes of the tidal constituents in the speed variations are more easily resolved than the phase angles, although they too display evidence of contamination from non-tidal forcings. This is demonstrated by step changes in phase between constituents with similar frequencies, rather than a smooth response (Zettler and Munk, 1975). However, the amplitude of $M_2$ is well resolved, and thus it provides a means for studying the upglacier propagation of tidal variations in motion through its amplitude admittance. We define the $M_2$ admittance as the ratio between the amplitude of the $M_2$ ice speed variation to the $M_2$ amplitude in the tide:

<table>
<thead>
<tr>
<th>$M_2$</th>
<th>2.05</th>
<th>16</th>
<th>0.96</th>
<th>-168</th>
<th>0.51</th>
<th>-150</th>
<th>166</th>
<th>-156</th>
<th>0.047</th>
<th>-166</th>
<th>0.022</th>
<th>-153</th>
<th>0.011</th>
<th>-177</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_2$</td>
<td>0.561</td>
<td>131</td>
<td>0.219</td>
<td>-32</td>
<td>0.092</td>
<td>-94</td>
<td>0.011</td>
<td>110</td>
<td>-168</td>
<td>0.004</td>
<td>-33</td>
<td>0.017</td>
<td>53</td>
<td>0.039</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.511</td>
<td>75</td>
<td>0.118</td>
<td>-96</td>
<td>0.231</td>
<td>-148</td>
<td>0.015</td>
<td>15</td>
<td>-144</td>
<td>0.015</td>
<td>-168</td>
<td>0.025</td>
<td>-167</td>
<td></td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.357</td>
<td>133</td>
<td>0.150</td>
<td>147</td>
<td>0.036</td>
<td>121</td>
<td>0.095</td>
<td>21</td>
<td>0.008</td>
<td>6</td>
<td>0.014</td>
<td>-128</td>
<td>0.010</td>
<td>93</td>
</tr>
<tr>
<td>$N_2$</td>
<td>0.342</td>
<td>41</td>
<td>0.242</td>
<td>-110</td>
<td>0.129</td>
<td>-28</td>
<td>0.120</td>
<td>-143</td>
<td>0.015</td>
<td>167</td>
<td>0.007</td>
<td>-68</td>
<td>0.018</td>
<td>176</td>
</tr>
<tr>
<td>$O_2$</td>
<td>0.247</td>
<td>108</td>
<td>0.199</td>
<td>-175</td>
<td>0.066</td>
<td>-174</td>
<td>0.176</td>
<td>166</td>
<td>0.023</td>
<td>125</td>
<td>0.016</td>
<td>-46</td>
<td>0.017</td>
<td>-131</td>
</tr>
</tbody>
</table>
This is shown in Figure 15 as a function of distance upglacier. There is an exponential upstream decay in $A_{M_2}$ with a characteristic damping length (perturbation damped to $1/e$ of its peak value) equal to 0.5 km, or about 1.5 times the centerline ice thickness at the terminus. This length may serve as a good proxy for the longitudinal coupling length over which longitudinal stress gradients are averaged in this region of the glacier (Echelmeyer, 1983; Kamb and Echelmeyer, 1986; Walters, 1989). This averaging length was used in internal deformation velocity calculations described earlier. According to the Echelmeyer and Kamb theory, strain rates on the order of $0.01 \text{ d}^{-1}$ should yield coupling lengths at about 1 ice thickness, which agrees nominally with our observed short decay length. However, the large contribution of basal motion at LeConte Glacier generally exceeds the assumptions used in their model, so the agreement cannot be expected to be exact.

**Diurnal variations**

Diurnal cycles exist in both the temperature and ablation rate. This likely influences water input to the glacier system, and therefore may affect glacier speed. However, resolving such forcing is difficult because diurnal tidal cycles contaminate any meltwater forcing due to their similar frequencies. Separation is especially difficult because of the broad spectral peak that characterizes the melt input, indicative of variable weather conditions (also noted by Walters and Dunlap, 1987).

In an effort to separate these two signals, we assume that $U_{\text{highfreq}}$ is a linear combination of two terms

$$U_{\text{highfreq}} = U_{\text{tide}} + U_{\text{melt}}$$

(5)
where the first term on the right accounts for motion driven by the tide, and the second represents meltwater-forced motion, plus noise. We then assume that $U_{\text{tide}}$ can be prescribed by the admittance of the $M_2$ constituent as applied to the other constituents:

$$U_{\text{tide}} = \sum_{i=1}^{6} A_{\text{tide}}^i \cos(\omega_i t - \varphi_{\text{exp}}^i)$$

(6)

where $\varphi_{\text{exp}}$ is the expected phase angle for the $i$th constituent if it is assumed to have the same phasing relative to its tidal constituent as $M_2$ does. Thus, we assume that the relative admittance of each of the six constituents is equal to the well resolved admittance for $M_2$. Tests with synthetic data have shown that this admittance transfer function (Eqn. 6) is effective for separating a melt signal from a tidal one (see Appendix II).

This admittance transfer function (Eqn. 6) was subtracted from the high frequency time series (Eqn. 5) to obtain $U_{\text{melt}}$, which is shown in Figure 16. These results show that melt-driven variations in horizontal motion are best developed at the upstream markers D through Bend. However, the speed of most markers shows a correlation with ablation rate. For example, the slowdown observed on day 125 is a result of a snowstorm (J.D. 122-123) that deposited ~30 cm of snow on the glacier, effectively shutting down meltwater production.

The average amplitudes of the melt-forced variations in speed were estimated from each $U_{\text{melt}}$ series. A step change in the peak-to-peak amplitude occurs between markers C and D; markers A through C have amplitudes ~50 cm d$^{-1}$ while markers upstream have smaller amplitude variations of only ~20 cm d$^{-1}$. A plausible explanation for this step change is the confluence of the adjoining unnamed glacier with LeConte between markers C and D.

To identify the lag between surface ablation and ice speed, we employ the method of cross correlation because of its ability to correlate time series with variable peak timing (in all cases the ablation rate peaks precede increases in speed) (Table 4). The statistical significance
of the correlations was small when calculated over the entire interval; this indicates that the phase lag between the two variables is not constant. However, by breaking the time series into two intervals (day 124-135 and day 135-154) on Julian Day 135, the correlation between the variables for each interval and each marker is greatly improved (Table 4 and Fig. 16). Day 135 thus marks a change in the ice speed response time to melt forcing; the average response time is reduced from 23 hours to 10 hours. Prior to day 135, the phase lag generally increases upglacier, but, after day 135, it is more variable, exhibiting no obvious trends.

Table 4. Cross correlation between \( U_{	ext{melt}} \) and ablation rate. The analysis was performed over the entire record, and again with a split on day 135 when an apparent change in the subglacial hydraulics took place.

<table>
<thead>
<tr>
<th>Markers and observation dates</th>
<th>Total Record</th>
<th>J.D. &lt;135</th>
<th>J.D. &gt;135</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase lag (hr)</td>
<td>C</td>
<td>Phase lag (hr)</td>
</tr>
<tr>
<td>A/A*(124-130)(135-143)</td>
<td>NA</td>
<td>NA</td>
<td>21</td>
</tr>
<tr>
<td>B/B*(124-129)(135-154)</td>
<td>NA</td>
<td>NA</td>
<td>21</td>
</tr>
<tr>
<td>C(124-146)</td>
<td>21</td>
<td>0.49</td>
<td>21</td>
</tr>
<tr>
<td>D (125-154)</td>
<td>12</td>
<td>0.22</td>
<td>21</td>
</tr>
<tr>
<td>F (125-153)</td>
<td>3</td>
<td>0.21</td>
<td>9</td>
</tr>
<tr>
<td>G (125-152)</td>
<td>9</td>
<td>0.20</td>
<td>12</td>
</tr>
<tr>
<td>Bend (128-144)</td>
<td>21</td>
<td>0.10</td>
<td>24</td>
</tr>
</tbody>
</table>

Short-term variations in vertical motion

We next consider the vertical position, \( z(t) \), of each marker over the same time scales as those discussed in regards to horizontal speed. For each marker, down-glacier movement was removed by subtracting a linear or quadratic representation of the local glacier surface from the \( z(t) \) series (Fig. 5). These surfaces were determined by airborne surface profiling (Fig. 17) (Echelmeyer, unpublished data, 1999). The resulting series were then analyzed via the same procedures as used for \( U(t) \).

Low frequency variations

The low frequency series of vertical displacement (Fig. 18) display dissimilarities between markers. Markers D through G exhibit fluctuations with relatively small differences
in peak timing. Here, the large peak appears to be a response to precipitation. In contrast, fluctuations in vertical position at marker A/A* are approximately in phase with the tidal amplitude; minimum surface elevation coincides with minimum tidal amplitude. The existence of a peak in surface elevation is suggested during the gap between A and A* at a similar time as the maximum tidal amplitude. Markers B and C do not show any correlation with either the tidal amplitude or precipitation. A mixture of these two forcings may be responsible for the glacier response, or the depicted trends may be a result of imperfect surface detrending.

Only the time series D through G resemble the low frequency time series of horizontal motion. After rain, the glacier surface is uplifted, then drops upon the initiation of upwelling. Analogous to extra slowdowns, the low frequency vertical series often show drops in surface elevation which are greater than the original uplift. As there is no longitudinal compression during the survey (Fig. 8), we may possibly attribute these variations to changes in basal water storage.

**High frequency variations**

Harmonic analysis of \( z_{\text{highfreq}}(t) \) shows that semi-diurnal tidal forcing of surface elevation exists only at the markers closest to the terminus (Fig. 19, A/A*, B/B*). However, the M\(_2\) response is small, with a 9% ROV for marker A/A* and a 4% ROV for marker B* (Table 5). The peak-to-peak amplitude of the M\(_2\) variation at marker A/A* is on the order of 13-18 cm, and the phase lags that of the tide by approximately 90°, such that the maximum surface elevation follows the high tide by ~3 hours. These Semi-diurnal vertical fluctuations are damped upglacier even more quickly (\( L_v = 0.3 \) km) than those found for the horizontal motion (Fig. 15).

**Table 5.** Harmonic analysis of vertical position. The variance reduction is shown for the M\(_2\) constituent as well as the combined reduction for diurnal constituents K\(_1\) and O\(_1\).
<table>
<thead>
<tr>
<th></th>
<th>ROV</th>
<th>(M₂) ROV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tide</td>
<td>7.5%</td>
<td>81%</td>
</tr>
<tr>
<td>A/A*</td>
<td>28%</td>
<td>9%</td>
</tr>
<tr>
<td>B/B*</td>
<td>31%</td>
<td>4%</td>
</tr>
<tr>
<td>C</td>
<td>22%</td>
<td>1%</td>
</tr>
<tr>
<td>D</td>
<td>17%</td>
<td>0%</td>
</tr>
<tr>
<td>F</td>
<td>21%</td>
<td>0%</td>
</tr>
<tr>
<td>G</td>
<td>6%</td>
<td>1%</td>
</tr>
</tbody>
</table>

The admittance transfer function (Eqn. 6) cannot be applied to \( z(t) \) because the semi-diurnal tidal constituent \( M₂ \) does not dominate the signal. The diurnal nature of the signal (Fig. 19) in the absence of any strong diurnal tidal forcing suggests meltwater forcing exists. In fact, for markers A through F the ROV by diurnal constituents \( K₁ \) and \( O₁ \) is significantly greater than the ROV in the tide for these same constituents (Table 5). The peak-to-peak amplitude of these diurnal variations is fairly constant (8-12 cm). Although close to our limit of uncertainty, these variations are not an artifact of optical surveying; they are present both in conditions of substantial precipitation as well as during times of clear weather. Additionally, we observed no longitudinal compression (Fig. 8), (as averaged over two day intervals), during the study period, so the variations in \( z \) are not due to changes in speed. Anomalously large diurnal uplifts often follow rainfall.

**Discussion**

In this discussion we interpret results of the velocity analyses, examining the processes driving velocity variations at LeConte Glacier. In cases where similar studies have been completed on Columbia Glacier, we compare our results with these. The discussion considers only the terminal reaches \( (0<ξ<7) \) of the glacier and does not apply to the upper glacier where different processes may be controlling the dynamics.

**Semi-diurnal variations**

Harmonic analysis has shown that ice speed varies 180° out of phase with tidal stage, such that maximum speed is achieved at low tide. Our results indicate that a 1.5% variation in
sea level (~3.0 m of 171 m mean depth) causes a 5.5% fluctuation in speed (~1.5 out of 27 m d\(^{-1}\) mean speed, marker A/A*). Equivalently, this may be expressed as a 0.5 m d\(^{-1}\) per m of tide. Tidally driven variations in speed have also been found on Columbia Glacier, where Meier and Post (1987) reported a 4% variation in speed forced by a 1% fluctuation in water level, or a 0.2 m d\(^{-1}\) fluctuation per m of tide, which is about half that observed at LeConte Glacier. Walters and Dunlap (1987) showed that the speed at Columbia Glacier is also nearly 180° out of phase with the tide. The similarity between the magnitude and phase of tidal forcing at the two glaciers suggests that high frequency variations in velocity are governed by the amplitude of the tidal fluctuations regardless of terminus geometry (slope, thickness, water depth), because the geometry is quite different between the two glaciers.

These semi-diurnal variations may be explained by a time-varying hydrostatic force imbalance at the terminus (Walters and Dunlap, 1987; Walters, 1989), where the ocean water column acts as a dam with a time dependent height. Maximum restraint from this dam occurs at high tide, when observed speeds are minimum. The variations in speed further depend on the amplitude of variations between high and low tides, being best developed when the tidal amplitude is maximum (Fig. 4). If, instead, the tide were to cause time varying pressurization of the basal hydraulic system, then one would expect zero phase lag between the tide and speed, assuming that basal motion is proportional to the basal water pressure. However, the presence of semi-diurnal variations in surface elevation at the terminus show that the water column not only acts as a dam, it also pressurizes subglacial water (these vertical variations are not due to longitudinal compression). However, these pressure fluctuations must be smaller than the forcing from the water dam or an in phase relation between the tide and ice speed would result.
We calculated an \( e \)-folding length for semi-diurnal perturbations of about 0.5 km. This value is much smaller than the 2 km \( e \)-folding length at Columbia Glacier. The rapid decay of high frequency perturbations at LeConte Glacier is likely attributable to the steep surface slope and rapid upglacier thickening, which create large longitudinal strain rate gradients (Kamb and Echelmeyer, 1986). Columbia Glacier is less steep, and has much smaller thickness and longitudinal strain rate gradients, and therefore a longer coupling length.

A glacier with a floating terminus will display strong semi-diurnal (\( M_2 \)) vertical variations (e.g. Jakobshavn Isbræ; Echelmeyer, unpublished), while a well grounded glacier will not exhibit \( M_2 \) surface elevation fluctuations. The rapid decay (\( L_v = 0.3 \) km) of semi-diurnal vertical variations implies that LeConte’s terminus must be grounded, but near floatation. Our result constrains this nearly floating region to a longitudinal distance of \( \sim 300 \) m upglacier from the terminus, which is equal to the characteristic decay length for vertical \( M_2 \) fluctuations (Fig. 15).

Diurnal variations

Diurnal variations in speed with amplitudes ranging from \( \sim 10 \) to 70 cm d\(^{-1}\) (up to \( \sim 5\% \) of the mean speed) are present over the entire terminus region; at the terminus they are about half the magnitude of tidally forced variations. While semi-diurnal variations in speed owe their existence to time varying seawater pressure, the observed diurnal fluctuations are driven by changes in water input from surface ablation. Diurnal periodicity is best developed upglacier from the tidally influenced region, at Bend/ Gate, but the largest absolute amplitudes occur at the terminus, where the cumulative basal water flux is maximum. The response time for these velocity variations is highly variable, both spatially and temporally; it changes in a step-like fashion most likely as a response to re-organizations of the subglacial drainage network. Similar fluctuations were observed at Columbia Glacier; Walters and Dunlap (1987)
estimate that melt-driven variations in speed are approximately one third the magnitude of tidally forced semi-diurnal variations. These authors report a more constant response of ice speed to meltwater forcing, with an average ice speed peak 7 to 8 hr after the peak in insolation. Unfortunately, the methods used to estimate melt-forced fluctuations in speed at Columbia and LeConte Glaciers are dissimilar, and they are both inadequate. However, both analyses show that these fluctuations exist, and are smaller than those driven by the tide.

Surface elevation also varies diurnally; the largest fluctuations are again found near the terminus, where the average amplitude is about 15 cm. This is a likely indication that basal water storage fluctuates diurnally (Iken and others, 1983). Support for water storage fluctuations also stems from anomalously large diurnal surface elevation variations after precipitation events. This may indicate that water discharge is the limiting process in determining diurnal fluctuations in water storage, similar to the findings at Columbia Glacier (Kamb and others, 1994; Meier and others, 1994). Our results show that storage fluctuations are associated with fluctuations in both horizontal speed and surface elevation.

On day 135 (May 15) an abrupt change occurred in the timing of the response of the speed to ablation. This event also coincides with a period of significant upwelling, although no rain had fallen for three days. At the same time, there was a large spike in the speed at Bend and marker F, while markers C and Gate both slowed (Fig. 4a). Perhaps most notably, the largest calving event of the study interval occurred on this day (see Table 6). These events followed a period of high ablation, that may have triggered a major reorganization of the basal hydraulic system. An alternate explanation is that the responses were forced by calving, but this is unlikely, as other large calving events do not produce noticeable changes in the ice speed or the response time to melt forcing. Taken together, these occurrences seem to imply that the short term dynamic behavior of this tidewater glacier is more dependent on basal processes
than changes acting at the terminus, except at a semi-diurnal scale, where forcing originates only at the terminus.

**Low frequency variations**

Speed-ups of 5-13% of the mean and lasting about three days were observed after periods of precipitation. The magnitude of these speed-up does not always vary in direct accordance with the magnitude of the precipitation. It can be expected that the properties of the basal hydraulic system must change frequently as a result of rapid basal motion (Willis, 1995), and these changes may account for the indirect relationship between the magnitude of speed-ups and storm events on LeConte Glacier. Similar results were found at Columbia Glacier (Fahnestock, 1991; Kamb and others, 1994; and Meier and others, 1994). These authors found that peaks in speed were centered on peaks in water storage (as indicated by a proxy record of discharge).

On LeConte Glacier, the timing of the speed-up on day 145 suggests that it is related to an increase in water storage; the peak of the speed-up occurs between a major rain event and a period of substantial upwelling (**Fig. 11**). At some markers, an “extra slowdown” follows this speed-up, which also argues for a water storage control. However, this slowdown is not observed at each marker, implying a poorly connected subglacial hydraulic system (Fahnestock, 1991; Kamb and others, 1994). The asymmetric shape of the surface elevation perturbations provides strong evidence for varying water storage (see **Fig. 18**), with discharge as the rate limiting control on storage.

**Relation between horizontal and vertical motion**

Phase relations between horizontal and vertical motion provide information on the processes controlling basal motion. If the pressure of basal water is controlling basal motion (and thus, the overall motion of this glacier), then the maximum horizontal speed (**Fig. 11**) should occur synchronously with the maximum vertical speed (times of maximum slope in $z(t)$).
series, **Fig. 18**), as the pressure is maximum at this time. If, instead, the peaks in horizontal speed and vertical displacement are in phase, then water storage may be important in determining basal motion (Paterson, 1994, p. 145-51). We examined both the high and low frequency horizontal speed and vertical position time series to determine the role of basal water on ice motion.

Cross correlation between $U_{\text{highfreq}}$ and $z_{\text{highfreq}}$ appears to indicate that the maximum surface elevation lags the maximum speed by three hours, a suggestion that water pressure is responsible for variations in motion. However, the correlation is poor ($C = 0.33$), and an inspection of the two time series shows that phasing between the peaks varies throughout the study interval (**Fig. 20**). We interpret this observation as an indication of mixed forcing from both water pressure and storage. The peak in horizontal speed often occurs after the peak in both vertical speed and vertical displacement (e.g. day 143), suggesting a delayed response of ice speed to forcing from both pressure and storage.

The motion response to pressure and storage fluctuations over longer time scales is also variable. An inspection of **Figures 11 and 18** shows that marker G exhibits a pressure-driven response, while just downstream at markers D and F the response appears to be due to fluctuating water storage. Closer to the terminus, markers B and C show what is likely a mixed response to forcing from both variables. All these results indicate that the speed is dependent on both subglacial water pressure and the volume of stored water there. Generally, the pressure influence increases upglacier. The large basal speeds in the terminus region, and the large velocity gradient present between Gate and marker A are in agreement with this interpretation. Reorganization of the basal drainage system should occur most frequently where basal speeds are highest, giving more variable forcing modes. Upstream, where basal motion is much reduced, we expect a more classic pressure driven response.
**Relative magnitudes of the variations**

Our results show that the relative forcing of the horizontal speed at semi-diurnal, diurnal, and low frequency scales is spatially and temporally dependent. At the terminus, tidal forcing is the strongest, but this is rapidly damped in the first 0.5 km upglacier. As the semi-diurnal variation decays, the diurnal melt-driven variation becomes more dominant. Thus, near the terminus the magnitude of tidally-forced variations are greater than those caused by fluctuations in input and/or storage, but the basal water variations affect the entire 7 km region above the terminus. Precipitation events can cause fluctuations in speed larger than either tidal or melt forcing, as evidenced by the nearly complete removal of the tidal fluctuations during the speed-up around day 145 (e.g. markers A* and B* in Fig. 4a). The relative magnitudes of these three horizontal velocity variations are similar to those observed at Columbia Glacier. At both glaciers, tidally driven variations are the largest; they range from 0.2 to 0.5 m d\(^{-1}\) per meter of tide (Walters and Dunlap, 1987). The diurnal variations are smaller than the tidal fluctuations in both cases, about a third as large at Columbia (Walters and Dunlap, 1987) and half as large at LeConte. At both glaciers, precipitation forcing is dominant when present, removing the other time scale fluctuations.

**Seasonal variations**

Our measurements of velocity over the 90+ day period from the pre-melt season to the end of the summer suggest that velocity in the terminal region does not exhibit seasonal variations. This result contrasts the observed seasonal velocity variations observed at Columbia Glacier, where Krimmel, (1997); Meier and Post, (1987); and Krimmel and Vaughn, (1987) report a maximum speed in early spring, and a minimum in early fall. On Columbia Glacier, the difference between the maximum and minimum speed averages about 2.5 ±1 m d\(^{-1}\) out of a mean speed of 10 – 15 m d\(^{-1}\).
If LeConte and Columbia Glaciers behaved similarly, we would have expected to observe a decrease in speed from May to August at LeConte Glacier. Given the high speeds in the terminus region (25 m d$^{-1}$), a slowdown of about 5 m d$^{-1}$ was expected if such seasonal variations were present. However, no such variation is apparent (Table 1, Fig. 9). The lack of seasonal variations in speed and continuous rapid flow (due primarily to basal motion) indicates that the bed must be well lubricated year round, even in the absence of surface water input. The origin of this water may stem from basal motion itself. An estimate of the heat produced by friction at the bed can be estimated from the relation $q_f = U_{\text{bed}} \tau_b$ (Paterson, 1994). With a driving stress of 2-2.5 kPa, and a basal sliding speed of 20 m d$^{-1}$, this yields a (maximum) estimate of 1.5 cm of daily melt production at the bed. This alone could provide enough water to allow continuous rapid motion regardless of the season.

Abundant basal melt does not, however, provide an explanation for the lack of a spring speed-up when surface meltwater flux increases or the lack of the summer slow down that was found at Columbia Glacier. This may imply that pressures in the terminal reaches are close to overburden, and thus, that basal drag is minimal. Then the major source of restraint is provided by the valley walls. However, this causes a problem when trying to explain diurnal variations in speed. It may be that basal drag is small but variable, while the drag against the valley walls is steady, but larger, providing the dominant restraint. Then the minimal basal restraint gives rise to the rapid motion, while small fluctuations in this drag result from fluctuating water input rates and drive diurnal variations in speed.

A separate mechanism which may explain both continuous rapid flow since the onset of retreat and diurnal variations in speed may be retreat from the terminal moraine. Fischer and Powell (1998) suggest that, for a stable tidewater glacier, the terminal moraine provides the dominant restraint to flow by providing a continuous ‘backpressure’ (longitudinal stress
gradient). Indeed, accelerating flow has been observed only after the initiation of retreat at both Columbia Glacier (Krimmel, 1997). Velocities have been steadily increasing since first measured at LeConte Glacier, one year after the initiation of retreat. These observations demonstrate the importance of the moraine in restraining ice flow. As the glacier retreats from its terminal moraine, this restraint is removed, driving a long-term velocity increase, in accordance with observations. Diurnal variations in speed may still be driven by diurnal changes in basal water pressure or volume in this conceptual model.

More support for this hypothesis is found in the timing of the velocity maximum, or at LeConte, the lack of a maximum. When Columbia Glacier was still grounded on its moraine, the maximum speed at the terminus was out of phase with the maximum speed elsewhere (Krimmel and Vaughn, 1987). As the glacier retreated, this phase difference was reduced to zero, suggesting that the moraine governed the terminus ice speed in some fashion. Bathymetry measurements in LeConte Bay (Motyka and Hunter, unpublished data) do not indicate the presence of a large terminal moraine at the present terminus, and there is no phase difference between velocities in the terminus region. It thus seems likely that the removal of morainal restraint initiates increases in speed, which are accentuated as the retreat progresses into deep water. The mechanics of this process may be related to glacier buoyancy: because cliff heights remain fairly constant even as the ice retreats into deep water (Meier, 1997), the height above buoyancy is reduced, driving increases in speed according to the accepted basal motion theory (e.g. Paterson, p. 151). Additionally, the effective pressure is reduced since water pressures required for discharge increase with water depth. Both of these conditions have potential to drive increases in ice speed and rate of calving.

Conclusions
By performing short time scale observations of glacier motion in the terminus region of LeConte Glacier, we gained an understanding of the controls on motion, which are integral to tidewater glacier stability. Our measurements delineate short time scale forcing mechanisms, indicating that

- Near the terminus, an out of phase relationship exists between ice speed and tide stage, such that maximum tide corresponds to minimum speed. This is a result of a changing “backpressure” from the ocean water level at the glacier terminus.
- The amplitude of melt-driven diurnal ice speed variations is about half as large as the semi-diurnal variations, but they are more widespread.
- Low frequency variations in speed are 5-15% of the mean, and are forced by precipitation-driven changes in water storage and pressure. The forcing mode varies in time. No direct relation exists between the magnitude of the extra input and resulting speed-up, showing that the response depends on the state of the basal hydraulic system prior to the input event.
- No seasonal changes in speed exist in the lower 7 km region of the glacier. This is possibly related to the lack of a terminal moraine and large basal melt rates, which provide a continuous source of basal water. Continuous rapid flow may also be linked to retreat from the terminal moraine by changing the height above buoyancy in the terminus region, reducing basal traction and driving fast flow.

In the next chapter we use these observations of ice motion as a basis for an analysis of iceberg calving. The primary aim is an investigation of short time scale variations in calving, specifically identifying the role of ice velocity and glacier buoyancy as driving mechanisms for calving.
Chapter 3.

Short-term variations in calving at a retreating tidewater glacier: LeConte Glacier, Alaska

Introduction

Tidewater glaciers undergo cycles of slow advance and rapid calving retreat, which may be asynchronous with both variations in climate and the fluctuations of nearby glaciers (e.g. Post, 1975; Mann, 1986; Motyka and Beget, 1996). Rapid retreat may be the expression of an unstable response to a negative mass balance (Hodge, 1979), and often results in the disintegration of a significant portion of the ablation area. Disintegration proceeds through iceberg calving during times of retreat. Mass loss through calving is typically much greater than loss from surface melting. In contrast, termini of terrestrial glaciers normally respond in a more predictable manner to changes in climate; a negative mass balance causes terminus retreat with a magnitude proportional to the balance deficit (eventually).

The processes that initiate calving retreats are poorly understood, as are the mechanisms that initiate individual calving events. This is partly because most studies to date have focused on calving rates over seasonal and longer time scales (e.g. Brown and others, 1982; Meier and Post, 1987; Venteris, 1999), and only one glacier (Columbia) has been studied during the initiation of retreat.

Our current understanding of iceberg calving is derived from observations and theory for both floating and grounded tidewater glaciers. However, the different styles of calving in these two settings makes a universal calving law improbable (van der Veen, 1997). In this paper, we consider only grounded, temperate glaciers.

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Hughes (1992) developed a theoretical model for calving from grounded glaciers terminating in water of variable depth, with bending shear as a calving mechanism. In his model, an extensional bending stress develops at the ice cliff because the lithostatic stress is not balanced by the opposing sea water pressure. This bending facilitates calving through the development of an overhanging ice cliff and subsequent opening of transverse crevasses near the terminus. However, several assumptions in this theory degrade its applicability to retreating tidewater glaciers. Most notably, Hughes’ model applies only in cases where water depths are small, so that the glacier terminus is far from floatation.

Brown and others (1982) considered twelve Alaskan tidewater glaciers, and derived a relationship between the width-averaged annual calving rate and the water depth at the terminus. This relation has become widely accepted, although a physical reason for the relation remains unclear. The water depth hypothesis has not gone unchallenged. Sikonia (1982) showed that this relation failed to explain observations made at Columbia Glacier over seasonal time periods. Instead, he advanced a relationship between the centerline calving rate and subglacial discharge as estimated by the proxy discharge of a nearby stream. Pelto and Warren (1991) also dispute the water depth relation, even over annual periods, asserting that the relation may not be causal. They claim that increases in water depth may drive an increased calving rate, but the inverse is not necessarily true: increased calving rate does not always imply an increase in water depth. Van der Veen (1996) expanded this idea by suggesting there may be different mechanisms governing calving on steady state and rapidly retreating glaciers. He shows that the linear relation between calving rate and water depth, as suggested by Brown and others (1982), holds only for the eight glaciers in the data set that are near steady state.

Glacier buoyancy has also been advanced as a calving control, and must be an important parameter during calving retreats in deep water situations. Meier and Post (1987)
first suggested that the Columbia Glacier terminus retreats to a location where the effective pressure becomes positive, or, equivalently, where the glacier is well grounded. Van der Veen (1996; 1997) further developed this idea, suggesting that calving occurs as a glacier thins to a critical thickness above floatation. Qualitative observations (Echelmeyer and others, unpublished data, 1999) provide support for this idea. Tidewater glacier termini range between 40-60 m in height, regardless of whether they are advancing or retreating.

During the Columbia Glacier retreat, the ice speed increased synchronously with increases in calving (Krimmel, 1997). This decreased the amount of expected retreat, and forced rapid upglacier thinning (Venteris, 1997). These findings prompted investigations of the role of ice velocity (van der Veen, 1996; Hanson and Hooke, in press) and longitudinal strain rate (Venteris and others, 1997) in calving. Both ice velocity and longitudinal strain rate show strong correlation with calving rates over long time scales (seasonal or longer), but they have not been investigated over shorter time scales. If calving events are not stochastic in nature, but occur as a response to changes in variables such as ice speed, stretching, or buoyancy, then we may expect to find similar correlations at time scales spanning hours to weeks.

On LeConte Glacier we have investigated short term fluctuations in velocity. We use the methods and results of this velocity study discussed in Chapter 2 to examine variations in calving at hourly to monthly time scales. We do not address the initiation of calving retreats, rather we seek the processes which initiate individual calving events and promote calving during retreat.

LeConte Glacier is a grounded, temperate, tidewater glacier, located in southeast Alaska (Fig. 1). It has been in a state of rapid retreat since 1994, undergoing about 2 km of retreat and substantial thinning. The recent thinning rate (1996 to 1999), as measured by airborne altimetry (Echelmeyer and Harrison, unpublished data, 1999), is 2.4 m a\(^{-1}\) averaged
over the entire glacier. The value is much larger at the present terminus, where the glacier has thinned at least 120 m since 1996.

Our study interval spans approximately one month, over which the position of the glacier terminus continuously fluctuated by about 90 m total at the centerline (less at the sides), with a standard deviation of 10 m between measurements typically made 4 times daily. Based on pro-glacial hydrography, we believe that the water depth at the terminus is essentially constant through the region of terminus fluctuation. Therefore we can study the effects of varying other parameters on the occurrence of calving events, while water depth is held constant.

**Observations and methods**

Direct observations of calving are difficult for obvious reasons, and only one such study has been conducted prior to ours. Warren and others (1995) used bedrock shelves above the terminus of Glacier San Rafael, Chile, as viewing platforms for documenting short-term variations in calving. We used similar viewing platforms at LeConte Glacier to observe calving from 2 May to 4 June, 1999. The month of May was chosen because it has previously been observed to be a month during which the glacier length underwent seasonal changes.

From our camp (**Fig. 1b**), we compiled a qualitative, visual record of calving events. Additionally, we utilized time-lapse photography to measure the position of the terminus on a sub-daily basis. The two data sets each provide unique information on calving, and they also provide two independent means of determining the temporal characteristics of calving. The visual data documents individual calving events, but it is subjective and we cannot document specific calving events during times of darkness. In contrast, the time-lapse photography does not record individual events, but it documents the nightly change in terminus position. These
photographs were used in conjunction with the measured influx of ice to the terminus (see Chapter 2) to produce a time series of calving flux.

**Visual monitoring data**

Throughout the study, the timing and magnitude of all daytime calving events was recorded using a subjective magnitude scale ranging from 1 to 10. Magnitude 1 events represent small pieces of ice breaking off the terminus, while a magnitude 10 event is representative of a collapse across the entire width of the terminus. Calibration of the scale was quickly accomplished between all observers. In **Table 6** we list the time and magnitude of the twelve largest calving events observed during the study interval.

**Table 6. Large calving events.** The times of the 12 largest calving events observed during the study are listed with their respective magnitudes on a scale ranging from 1 to 10. The tide level is also given in parentheses.

<table>
<thead>
<tr>
<th>Time (d)</th>
<th>Magnitude</th>
<th>Time (d)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>126.58</td>
<td>9 (Rising)</td>
<td>145-146 (night)</td>
<td>9 (Falling)</td>
</tr>
<tr>
<td>133.65</td>
<td>10 (Falling)</td>
<td>147.24</td>
<td>10 (Low)</td>
</tr>
<tr>
<td>135.29</td>
<td>10+ (Low)</td>
<td>147.45</td>
<td>9 (Rising)</td>
</tr>
<tr>
<td>137.43</td>
<td>9 (Low)</td>
<td>147.65</td>
<td>9 (High)</td>
</tr>
<tr>
<td>139.67</td>
<td>10 (High)</td>
<td>150.54</td>
<td>10 (High)</td>
</tr>
<tr>
<td>143-144 (night)</td>
<td>10 (Falling)</td>
<td>153.53</td>
<td>10 (Rising)</td>
</tr>
</tbody>
</table>

The occurrence of massive calving events normally follows a repeatable sequence. These events are typically initiated by calving off the sub-aerial ice cliff. This is followed by submarine calving of the mid-portion of the terminus, then by calving of deep basal ice, which comes from large water depths. There is often a lag of several to tens of minutes between the sub-aerial collapse and the time that the submarine portions calve (Motyka, 1997). Buoyancy instability often lifts the tops of submarine bergs 40-60 m out of the water upon calving. There is rarely a down-fjord component of velocity in the emergence of these submarine bergs. On occasion, we observe submarine events without a subaerial collapse, but these events are much less frequent. The color of the icebergs indicates the original location of the berg when it was
attached to the terminus. Subaerial ice is characterized by air bubbles and a white color; deep-basal ice is a dark blue bubble-free ice, and the mid-section ice lies in between these end members.

**Time-lapse photography data**

Both time-lapse and aerial photography have been previously employed (see Krimmel, 1997) to measure both changes in the position of the calving front (\(dL/dt\), where \(L\) is the length of the glacier) and the near-terminus ice velocity, \(U_i\) (specifically, the width averaged speed at or near the terminus, e.g. Krimmel, 1987). These data are then used to derive the calving rate, \(U_c\) (e.g. Brown and others, 1982), defined as the difference between the width-averaged ice speed and the rate of change of glacier length:

\[
U_c = U_i - \frac{dL}{dt}
\]  

(7)

At LeConte Glacier, we measured the position of the terminus (\(dL/dt\)) using oblique time-lapse photography. At the same time, but on a different schedule, we surveyed velocities at or near the terminus to obtain \(U_i\), as described in Chapter 2. However, because we know the transverse profiles of ice thickness and velocity, we cast Equation (7) in terms of volume flux (see also Meier and others, 1980)

\[
Q_c = Q_{in} - Q_{out}
\]  

(8)

where each of the terms were both calculated across a central flux band as defined by the region visible in time-lapse images (about 75% of the total width of the glacier terminus; Fig. 2). The calving flux, \(Q_c\), which represents the cross sectional average calving speed, \(<U_c>\), multiplied by the cross sectional area of the terminus, \(S\),

\[
Q_c = <U_c>S
\]  

(9)

is calculated as the difference between the measured value of incoming ice flux, \(Q_{in}\), and the measured flux out, \(Q_{out}\) (in m\(^3\) d\(^{-1}\)). In addition to calving losses, \(Q_c\) implicitly incorporates
submarine melting at the terminus. The measured bathymetry and effective cliff height (Fig. 2) provide the necessary cross sectional area at the terminus,

\[ S = \int_0^w h(x) \, dx \]  

where \( x \) is the transverse coordinate across the mean terminus position, \( W \) is the width of the flux band, and \( h(x) \) is the effective ice thickness. An effective ice thickness is used to account for void space due to intense crevassing in the terminus region (Echelmeyer and others, 1991). On LeConte Glacier, the average cliff height is 60 m and the upper 30 m of ice contains approximately 25% void space. This leads to an average effective cliff height of 52.5 m above sea level, to which we add the water depth as measured about 200 m down fjord of the terminus, to obtain the effective thickness. Note that this value excludes any bottom crevassing.

The terms on the right hand side of Equation (8) are calculated using the relations

\[ Q_m = <U_{\text{def}}> \, S + \int_0^w U_{\text{bed}}(x) \, h(x) \, dx \]  

\[ Q_{\text{out}} = \int_0^w \frac{dL}{dt} \, h(x) \, dx \]  

where \( <U_{\text{def}}> \) is the cross sectional average velocity due to internal deformation (about 2 m d\(^{-1}\)) and \( U_{\text{bed}} \) is the basal motion. In the terminus region, \( U_{\text{bed}} \) is the primary component of ice flow (see Chapter 2), accounting for 80-90% of the surface motion.

The incoming ice flux is derived from velocities measured at markers located within 200 m of the terminus. We did not average this ice velocity over the width of the flux band. Rather, we scaled the transverse velocity profile (Fig. 3) to the measured speed of the centerline marker nearest the terminus at the time of each survey (A, A* or B*; see Chapter 2
for ice velocity analysis). We also removed the increase in speed induced by down-glacier movement through the large near-terminus velocity gradient, thereby approximating an Eulerian reference frame at a point 150 m from the mean terminus position in May. Calving loss of marker A* necessitated switching to marker B* for centerline speed measurements. This marker was located further upglacier and has a lower mean speed than A or A*. Therefore, when determining the centerline speed at our reference point using B*, we adjusted for the spatial velocity gradient between these two markers. Although we feel that these flux estimates are a better representation of glacier flow than previously used width averages, they still rely on two basic assumptions. We assume that the transverse velocity profile (Fig. 3) is steady in time, and we assume that the flow direction at the terminus is normal to the x-axis.

Two oblique-looking 35 mm time-lapse cameras (one with a 50 mm lens, the other a 100 mm lens) were used to determine $dL/dt$ in Equation (12). The two cameras were set up at approximately the same location above the south side of the 1999 terminus. Two cameras were used to maximize the possible number of photographs each day, and provide a backup in case of camera malfunction. The 50 mm lens had a wider field of view, therefore rescaling was necessary for a direct comparison with images from the 100 mm lens. Figure 6 shows two typical time-lapse images.

In each frame, the terminus position was obtained following photogrammetric techniques described by Krimmel and Rasmussen (1986) and Harrison and others (1992) (see Appendix III). The terminus position in each frame was then differenced from the previous frame to give $dL/dt(x)$. Rather than calculating a width-averaged terminus position, we integrated $(dL/dt, \times \text{the local effective thickness})$ across the channel to yield a time series of the outgoing ice flux (Eqn. 12). Two assumptions are made in this development. First, although observational evidence suggests that a submarine ice toe may develop and persist for
periods longer than our sampling interval (Motyka, 1997; Warren and others, 1995), we assume that the terminus fails vertically from the glacier surface to the bed between subsequent frames. Second, we assume that any ice flow outside the flux band contributes a negligible amount to the total flux, and our flux band is taken to represent the entire flux at the terminus.

**Analysis and Results**

Figure 21 presents the results of the photogrammetric flux analysis. In Figures 21a and b we show $Q_{in}$ over the survey period. Figure 21a has an expanded vertical scale to illustrate small variations in flux, while (b) is plotted at the same scale as $Q_{out}$ in Figure 21c. At this latter scale the flux in is nearly constant, while the short-term flux out varies substantially. The outgoing ice flux, $Q_{out}$, is positive during times of terminus advance, and negative during retreat. Figure 21d presents the calving flux, which is the difference between panels (b) and (c) by Equation (8). An average error bar is shown in each panel. Note that although there are large errors associated with a lack of control points in the photos (see Appendix III), there are large changes in $Q_{out}$ and thus $Q_{c}$, that exceed our estimated error bounds. While there are some discrepancies between the photogrammetric data set and the visual observations due to shortfalls inherent to each data set, the two data sets do correlate well for the most part (Fig 22).

The monthly average calving flux, $Q_{c} = 2.97 \times 10^{6}$ m$^3$ d$^{-1}$. Over the entire study, the 5 day average calving flux is nearly constant, varying by only small amounts about this monthly average. From the monthly average, we estimate that the mass loss from the glacier due to calving is about 15 times greater than the mass loss from surface melting; about 1.1 km$^3$ a$^{-1}$ ice is lost from calving, while only about 0.07 km$^3$ a$^{-1}$ is lost due to surface melt.

The magnitude of short-term variations in $Q_{out}$ is much greater than the variations in $Q_{in}$. In other words, the volume of ice lost in a calving event is much larger than any change in
ice flux introduced by semi-diurnal changes in ice speed. Because $Q_{in}$ is nearly constant, the calving flux is directly correlated with $Q_{out}$ via Equation (8) as shown in Figure 21. The spikes in the record of calving flux represent individual calving events and are not an estimate of a ‘calving rate’. To estimate a true calving rate, we must average $Q_c$ over time periods that encompass several major calving events, which is several days in our case. In this paper, we seek relationships between the major calving events and other measured parameters that may be influencing the degree and timing of these events. Thus we analyze the “noisy” curve of $Q_c$ shown in Figure 21d, and not the longer time (and smoother) averaged values.

**Incoming ice flux**

Over relatively long time scales (e.g. annual) the ice velocity at Columbia Glacier shows a strong correlation with the calving rate (van der Veen, 1996). This contrasts with our result shown in Figure 21. Over short time scales, there is little correspondence between calving events and changes in ice speed, even when considering the small changes in $Q_{in}$. Rather, ice is supplied to the terminus at a nearly constant rate, except for some minor variations forced by the tide, melt, and precipitation (see Chapter 2). Major calving events do not cause noticeable changes in the ice velocity, or flux in. This is shown in Figure 23 where $Q_c$ is punctuated by brief changes caused by calving followed by periods of re-supply lasting 2 to 3 days. Thus although $Q_{in}$ is nearly constant, it is a necessary component in calving, serving to re-supply lost ice.

**Longitudinal strain rate**

It has been proposed that changes in longitudinal strain rate cause changes in the rate of calving at Columbia Glacier over seasonal and annual time scales (Venteris and others, 1997). The basic idea is that fluctuations in stretching lead to changes in the rate of thinning, which is important in regulating calving. On LeConte Glacier, we have investigated the relation between short-term variations in longitudinal strain rate and calving. The longitudinal strain rates are
extremely high, with a mean value in the terminus region is about 2.5 a\(^{-1}\) (Fig. 8). They generally increase towards the terminus, except for an abrupt change in the last 200 m to the terminal ice cliff. The longitudinal strain rates between markers do vary with time, but the variations are small (Fig 24). However, the two day average of longitudinal strain rate does not show any correlation with the calving flux.

Glacier buoyancy

As already mentioned, some authors have proposed a relation between calving and the degree of buoyancy. Here we investigate the relations of various parameters that affect the floatation level of the glacier to calving events. Water depth, longitudinal stretching (resulting in thinning), basal water pressure and storage all may affect glacier buoyancy. We use the methods of harmonic analysis (Chapter 2), cross correlation, and qualitative comparisons applied to our calving data to investigate the influence of these parameters on specific calving events. The parameters that will affect buoyancy include the tidal amplitude, water input, water storage and water depth at the terminus. As in chapter 2, we perform these analyses over multiple time scales. Implicit to this discussion is that temperate glacier ice is too weak to sustain floating. This assumption is based on the fact that there are no observations of floating termini on temperate glaciers.

Semi-diurnal and diurnal forcing

The tide and meltwater input affect water depth and subglacial water storage and pressure. Therefore they will affect the floatation level at the terminus. Our time series of calving flux was not sufficiently sampled to satisfy the Nyquist criteria for a quantitative analysis of calving over semi-diurnal time scales. However, we can qualitatively analyze the calving events in terms of tidal forcing by comparing the timing of the largest calving events (magnitude ≥9, Table 6) to the tide. There were twelve major events during May, and they were equally distributed through the semi-diurnal tide cycle. Thus there was no correlation
between tidal stage and the likelihood of large calving events. This suggests that the semi-diurnal tide does not provide significant forcing of individual calving events.

The sampling interval of calving flux did, however, allow harmonic analysis of diurnal constituents. We found that the primary diurnal constituent in $Q_c (K_1)$, is nearly in phase with this component of the ablation rate (ablation rate, $\varphi_{K_1} = -20^\circ$; $Q_c$, $\varphi_{K_1} = -13^\circ$), suggesting a possible diurnal forcing of $Q_c$. However, cross correlation between the two time series shows little or no statistically significant correlation. We also considered the hourly distribution of daytime calving events greater than or equal to magnitude five (Fig. 25). There is no distinct peak in the timing of these events, but there is a possible broad peak between 10:00 and 12:00, suggesting a possible weak diurnal forcing of calving.

An additional test for diurnal forcing, is the cross correlation between $Q_c$ and $z(t)$ for marker B*, because a strong diurnal fluctuation exists in the surface elevation at this marker. The low correlation ($C = 0.33$) between the two records suggests that calving is not diurnally forced, but inspection of $Q_c$ (Fig. 26) shows the presence of some diurnal periodicity early in the record. These three tests suggest that there may be weak diurnal trends in the calving flux, but the origin of this forcing is not clear.

The cross correlation did show that five anomalously large surface uplift events, each lasting about a day and up to 20 cm in amplitude, coincided with or were followed by large calving events. These events are shown in the early portion of Figure 26. In this figure, the series of $Q_c$ has been shifted back in time by 0.25 d in order to illustrate the best correlation between the two series. At the beginning of the record the correlation is good, but it degrades over the latter third of the series after a several day period of continuous rain, which may have changed the basal hydraulic system. The largest two events follow rainstorms, and are separated by an upwelling event on day 143, suggesting that water storage may be important
for calving. The importance of surface elevation changes for calving is also shown in Figure 26, which shows the vertical motion of markers A and A*, which were located less than 100 m from the terminal cliff. Several large drops in surface elevation are associated with large calving events. We show all large calving events that occurred while surveying markers A and A*. The timing of the events are known to within 5 minutes, except for the final event, which was only recorded by the time-lapse cameras, and is poorly constrained. All of these calving events occurred just after surface drops, but all drops were not associated with calving events.

**Low frequency fluctuations**

Figure 27 shows calving and precipitation as a functions of time. We investigated the relation between precipitation and calving only after precipitation changed from snow to rain. Here, the $Q_c$ time series has been shifted back in time by 1 d to give the maximum correlation ($C = 0.12$). The correlation is poor, but inspection shows that substantial rain events sometimes correlate with calving events a day or so later (e.g. days 141, 144). However, several calving events occur during dry weather, and rain does not always result in calving. These two precipitation events also forced the anomalous uplift events discussed above. The speed up that occurred following the second (Fig. 21a; J.D. 144) precipitation event may be related to the calving event shown on that day, but again there were other active days of calving with little or no associated precipitation or speed increase.

Another low frequency fluctuation that may influence glacier buoyancy is the biweekly change in tidal amplitude. The tidal amplitude is taken as the average range between the two high and two low tides each day. The simplest analysis makes use of the daily sum of the visual calving data, as shown in Figure 28. A strong correlation ($C = 0.55$, with zero lag) between the tidal amplitude and this visual calving record suggests a link between the two. This warrants a more rigorous analysis using the calving flux series (Fig. 29). A maximum
cross correlation between $Q_c$ and the tidal amplitude was found at a zero to one day lag, such that calving follows the tidal amplitude. When the record was analyzed over the entire interval, the analysis gave a low correlation coefficient, $C = 0.13$ (Fig. 29a), with calving lagging the tidal amplitude by 1 day. If we remove the two large calving events that appear to be results of anomalous surface uplift (as driven by precipitation), $C = 0.23$ (Fig. 29b). An inspection of the time series shows that the correlation is indeed quite good. The time derivative of the tidal amplitude, gives the time rate of change of the tidal amplitude and shows an even better correlation with calving (Fig. 29c; $C = 0.25$). This indicates that the rate of change of the biweekly tidal range may be important in calving. Preliminary results from a similar analysis performed over three months also indicate that the tidal amplitude is an important mechanism in governing calving (Johnston, personal communication, 2000).

**Calving trends**

Figure 30 presents trends in calving over the entire study interval. The difference in final and initial terminus position over the study interval (Fig. 30a) shows that the central portion of the glacier advanced slightly while the margins did not change much. Thus it appears that, at least during 1999, May was not the month when seasonal retreat began, as we had suspected earlier. The location of the maximum advance (~50 m) corresponds to the location of the maximum velocity across the terminus. A comparison of the first and last images in the time-lapse sequence shows that the slope of the lower glacier changed substantially during May, with a decrease in surface slope throughout the study. The nominal cliff height remained relatively constant, and thus the ice behind the cliff must have thickened.

From early May to the end of August, the glacier terminus position remained fairly constant; the total retreat over the summer was only about 100 m. However, the terminus did
fluctuate by 20-50 m over short time scales as calving events occurred (Johnston, unpublished data, 2000).

**Figure 30b** shows the centerline terminus position as a function of time. The location of the terminus varied through a range of 90 m. This figure shows that terminus advance between calving events, is a slower process than retreat accomplished through calving events. There is an apparent weak periodicity of about 2 to 3 days between major calving events. This suggests that the glacier must replenish the recently removed area, thinning to a buoyant level required for calving before it calves again, but the evidence is not all that strong.

To test the water depth relation suggested by Brown and others (1982) at LeConte Glacier, we divided the average calving flux by the cross sectional area, $S$, to obtain $<U_c>$. Averaged over the month of May, the calving rate (Equation 9) is 19.5 m d$^{-1}$. Depth soundings near the terminus give an average water depth ($D_w$) of 171 m. Using Brown and others (1982) relation

$$<U_c> = 0.027 D_w$$

(13)
gives a predicted rate of calving of 12.5 m d$^{-1}$. The observed calving rate is approximately 35% greater than the value predicted by this relation. Similarly, the observed calving rate is much greater than that predicted by the calving relation proposed by Pelto and Warren (1991). An attempt was also made to investigate the relation between water depth and calving flux on a local scale. For this, we show the monthly average calving flux, $q_c$, across 50 m wide segments of our flux band, as a function transverse position across the terminus (Fig. 30c). If we instead plot $q_c$ as a function of water depth, the relation is linear ($r^2 = 0.90$). This linear trend results because the terminus position remained essentially constant during the study, and because the speed is consistently high in the deep water on the north side of the glacier (Fig. 3). These
results show that the water depth is important in calving, but variations in calving rate are not necessarily forced by changes in water depth.

**Discussion: processes controlling calving events**

We employ observations and measurements of calving events, ice velocity, terminus position, precipitation and the tide to discuss the occurrence of major calving events. To do this, we evaluate the role of incoming ice flux in calving, then continue by investigating some potential calving triggers that initiate calving by either flexing or floating the terminus. Because changes in glacier buoyancy may be the result of forcing from multiple processes, we can not expect to find a strong correlation between the occurrence of calving events and one single variable. Rather, we seek cases where individual calving events may be attributed to a specific, identifiable process, including the tide, meltwater or precipitation, longitudinal stretching, and the buoyancy instability that results when the subaerial cliff calves.

Retreating tidewater glaciers generally terminate in deeper water than stable and advancing tidewater glaciers, yet they all have similar cliff heights. Because of these deep water conditions, the termini of retreating tidewater glaciers are close to floatation, contrasting the situation at stable and advancing glaciers where submarine terminal moraines provide shallow water depths and backpressure stability. During retreat there is a lack of morainal backpressure, leading to increases in velocity and buoyancy in the terminal regions. Accelerating flow and large longitudinal stretching rates \( \dot{\varepsilon}_{zz} \approx 2.5 \, a^{-1} \), on LeConte Glacier result. As the glacier stretches, it thins substantially, and further approaches floatation. The occurrence of calving events may be purely stochastic, but it appears that the fractured temperate ice that composes these termini cannot sustain floatation, therefore calving occurs if floatation is reached. Before reaching floatation, small short-lived perturbations in buoyancy may also trigger calving.
Additionally, the buoyant glacier ice is susceptible to flexure which increases crevasse penetration depth. It may be possible for water filled crevasses to reach the bed in such highly extensional environments (van der Veen, 1998). Hughes (1992) also pointed out the importance of flexure, but the magnitude of bending predicted by his theory is insufficient to force calving at deep water terminating glaciers such as LeConte. However, Hughes’ theory may apply to the subaerial portion of the ice cliff, where forward directed subaerial calving is often observed.

Calving and velocity

Our measurements of \( U_i(t) \), \( dL/dt \), and \( S \), allow us to calculate \( Q_c \) from Equation (12). Figure 21 shows that \( Q_{in} \) was nearly constant over the study interval, but \( Q_{out} \) was highly variable. Even an examination of \( Q_{in} \) on an expanded scale (Fig. 21a) shows no correlation with \( Q_c \). Thus the variability in \( Q_{out} \), and the constancy of \( Q_{in} \) lead to a calving flux that tracks short-term changes in terminus position. The correlation is largely numerical, with little significant physical basis. A similar argument applies to the apparent correlation between the ice velocity and calving rate over annual time scales (van der Veen, 1996, p. 380-381). Annual terminus changes are small compared to the annual ice velocity at the terminus. The time averaging interval is therefore critical to interpretations of changes in calving flux (or, equivalently, calving rate). Our results suggest that monthly time averaging is best suited for interpreting the importance of incoming ice flux in governing the calving flux.

The short duration of our study necessitates using the visual calving data to investigate the occurrence of specific calving events. We found no evidence for a link between calving and ice velocity. Calving events were well distributed throughout tidally forced semi-diurnal cycles in speed. That is, the likelihood of a major calving event is not maximized at low tide, when the near-terminus ice velocity is maximum. Additionally, while calving events may
show weak diurnal periodicity, the timing of the diurnal peaks for melt-driven motion (Chapter 2) and calving are dissimilar, suggesting that diurnal velocity peaks are not responsible for any diurnal component of calving. On hourly time scales, over which calving events occur, large calving events do not alter the surface velocity, and semi-diurnal variations in speed remain unperturbed throughout periods of heavy calving. Thus, although large calving events may reduce backstress, no concurrent change in velocity is apparent, indicating that either the change in backstress is localized, or it is averaged out over the longitudinal coupling length of the glacier (0.5 km, see Chapter 2). Alternatively, the glacier terminus may be decoupled from the rest of the glacier through heavy surface crevassing and possible bottom crevassing, preventing upglacier transmission of longitudinal stress variations. The large drop in longitudinal strain rate at the terminus suggests this is the case.

Both LeConte and Columbia glaciers undergo seasonal variations in length. The length of Columbia Glacier varies such that maximum occurs in late spring, approximately 3 months after the maximum speed (Krimmel, 1997). The glacier retreats through the summer to a position of maximum retraction about three months after the minimum speed is observed; re-advance follows through winter and early spring. Similar seasonal variations in length occur at LeConte Glacier. Our time lapse record through the winter of 1998-99 shows that maximum length occurred during spring; minimum length in early winter.

Seasonal changes in glacier length have been attributed to seasonal variations in speed at Columbia Glacier (Krimmel and Vaughn, 1987). On LeConte Glacier, where ice influx is steady, this is not the case. By Equation (7), seasonal changes in calving rate must then control the length. Therefore, we investigate processes other than ice speed that may control the frequency and magnitude of calving events.

*Floatation and flexure triggers*
To date, there are no observations of floating temperate tidewater glaciers, however retreating tidewater glaciers are normally close to floatation. This leads to the assumption that calving occurs as the glacier nears floatation. To estimate the floatation level we assume that subglacial water pressure is tied to sea level. Then the average height above buoyancy, \( H_b \), at the terminus is

\[
H_b = H - \frac{\rho_w D_w}{\rho_i}
\]  

(14)

Here, \( H \) is the cross sectional average effective ice thickness (220 m; corrected for near surface voids, after Echelmeyer and others, 1991), \( D_w \) is the cross sectional average water depth (170 m), \( \rho_w \) is the density of seawater, and \( \rho_i \) is the density of ice. For a narrow, deep glacier, such as LeConte, the terms in this expression must be evaluated as cross-sectional averages in the near terminus geometry as hydrostatic equilibrium will not hold locally. This gives a height above buoyancy of 25 m (\( \sim 2 \times 10^5 \) Pa), assuming no internal voids.

In the case of a steeply sloping tidewater glacier, like LeConte, the basal hydraulic system may actually be subject to water pressures that are greater than hydrostatic pressure just upstream of the terminus. These conditions may extend quite close to or reach the terminus (e.g. Vieli and others, in press). Thus, water pressure can be important in accurately estimating the height above buoyancy, when the terminus is highly buoyant. Equation (14) gives a maximum value of \( H_b \), and it is likely that LeConte Glacier is actually closer to floatation than our stated value. If we couple this conclusion with the observation that temperate tidewater glacier termini have not been observed to float, then small perturbations in buoyancy and the associated changes in longitudinal stresses may possibly lead to calving. This is supported by the observations by Meier and others (1994) and Kamb and others (1994), who found that borehole water pressures 5 km upstream from the terminus of Columbia Glacier can change.
rapidly and provide nearly floating or floating conditions; they measured short-term water level fluctuations of 20 to 30 m. If such changes are widespread at the terminus of a tidewater glacier, they may temporarily bring extensive areas of the glacier to floatation, and thus induce calving. We will investigate some of the possible processes that may be acting as short lived triggering mechanisms for calving events in the sections that follow.

**Longitudinal stretching**

It has been postulated that thinning is critical for rapid calving (van der Veen, 1996; Venteris and others, 1997). On LeConte Glacier there is little or no morainal backpressure (Fischer and Powell, 1998), longitudinal stretching rates are large, and the ice stretches and thins until it fails. This failure appears to occur on or near floatation. Figure 30b shows that periods of terminus stability (as ice flows into the terminus) are followed by rapid changes in length (calving events). A calving event may increase ice thickness by up to 10 m and stable periods between calving events generally last 2 to 3 days. During these stable periods, as stretching re-thins the ice, the glacier approaches floatation. If short-term triggers do not bring the glacier to temporary floatation and cause a calving event, longitudinal stretching will thin the ice and cause calving.

**Tidal forcing**

The occurrence of calving events does not show a strong correlation with the semi-diurnal tide stage, as a buoyancy mechanism implies. Observations at Glacier San Rafael (Warren and others, 1995) have found only weak correlations between the semi-diurnal tide and calving and a study at Columbia Glacier found no correlation with semi-diurnal tide stage (Qamar, 1988). Tidally forced semi-diurnal variations in vertical position near the terminus are small, and decay rapidly (within ~300 m of the terminus; see Fig. 15), indicating that the tide is not causing large changes in basal water pressure. Thus the semi-diurnal tide alters the buoyancy of the glacier only through changes in geometry (thickness and water depth in Eqn.
Buoyancy changes that result from a 5 m water level variation are small, therefore a weak, or non-existent relationship between calving events and tide stage can be expected.

However, over longer time periods, as shown in Figures 28 and 29, there is a correlation between the bi-weekly tidal amplitude and calving. Periods of greater than average daily tide range appear to be more effective at weakening the terminus and forcing calving events than periods of below average tide range. This may be related to terminus flexure at times of maximum tidal amplitude.

The correlation between the low frequency component of vertical motion at marker A/A* and the tidal amplitude is shown in Figure 31, where the tidal amplitude has been shifted back by 2 days for maximum correlation. This suggests that water storage increases when the tidal amplitude is large. An increase in water storage requires an increase in basal water pressure (Iken, 1981), thus there is an increase of the buoyancy in the terminus region. Therefore, the bi-weekly variations of the tidal amplitude may cause buoyancy perturbations driven by basal water pressure. Pressure forced buoyancy increases, especially when coupled with concurrent flexure, appear to be large enough to increase the frequency of calving events.

**Meltwater and precipitation forcing**

Our analyses of meltwater and precipitation forcing for calving showed that variations in surface elevation were often associated with calving events (Fig. 26, 27). Two uplift events follow heavy precipitation, others follow periods of abnormally large melting. Periods of upwelling separate most uplift events. As there was no longitudinal compression in the lower reaches of LeConte glacier during the study (Chapter 2), the uplift events are likely a result of increases in water storage (Iken and others, 1983) and the associated buoyancy increase.

We may then expect a correlation to exist between local maxima in water storage and the occurrence of large calving events. Therefore, we estimated changes in water storage as
shown in Figure 32, where each bar represents the daily difference between water input and output (Chapter 2). When formulating this index, we considered above base level surface ablation and precipitation as inputs, and any visible upwelling as an above base level output, ranking the magnitude of these three parameters from 1 to 5. We acknowledge that this storage index is crude, mainly because water outflow is difficult to quantify and is therefore highly subjective.

Figure 32 shows no link between storage maxima and calving, rather there is a correlation between abrupt changes (either increases or decreases) in water storage and calving events. This correlation suggests that, in addition to triggering calving by floatation of local areas of the terminus, flexure of the nearly floating area behind the terminus may be a common trigger for calving events at LeConte Glacier. In this case, a buoyant glacier is a prerequisite for the flexure trigger. A closer examination the timing of this relationship is shown in Figure 26, where we show that all of the large calving events that occurred while surveying marker A/A* coincide with or are immediately preceded by large, rapid surface elevation drops up to 30 cm in amplitude. The flexure associated with these rapid drops in surface elevation is likely associated with changes in water storage or pressure, and must serve to propagate fractures and initiate calving.

Effective pressure

Only weak correlations exist between calving events and individual processes because multiple processes can effect the buoyancy and flexure of LeConte’s terminus by fluctuations in terminus geometry (H and \(D_w\), Eqn. 14) and water pressure. If it were possible to account for both pressure and geometric changes with one variable, a strong correlation with calving may result. The effective pressure, \(p_{\text{eff}}\), which is defined as the difference between ice overburden pressure and basal water pressure, accounts for such changes, and may be
important in calving. The basic idea is that the terminus stabilizes when \( p_{\text{eff}} > 0 \) and retreats as \( p_{\text{eff}} \to 0 \). This was first proposed by Meier and Post, 1987.

Extreme crevassing prohibits measurements of basal water pressure near the termini of retreating tidewater glaciers, so the height above buoyancy is normally substituted as a proxy for effective pressure (Sikonia, 1982; van der Veen, 1996). However, retreating tidewater glaciers may have poorly connected hydraulic systems and be quite steep. Therefore basal water pressures may exceed sea level pressure, even close to the terminus (e.g. Vieli and others, in press). In cases such as this, the height above buoyancy is not necessarily a good proxy for effective pressure.

Additional strength that perturbations in effective pressure may be possible triggers for individual calving events follows by an extension to longer time scales. Effective pressure has been previously suggested as an important factor for calving rates over seasonal time scales at Columbia Glacier (e.g. Sikonia, 1982; Fahnestock, 1991). We now consider the validity of effective pressure as a factor influencing seasonal variations in calving at LeConte Glacier.

**Seasonal variability in calving**

Rapid calving appears to require a nearly floating terminus (Sikonia, 1982; Meier and Post, 1987; van der Veen, 1996). As the terminus approaches floatation, vertical flexure of the terminus and short-term variations in effective pressure may trigger calving events. We attempt to extend this postulate to seasonal time scales by arguing that calving rates increase when effective pressure is low and variable, and decrease when effective pressure is higher and steady. Seasonal cycles in \( p_{\text{eff}} \) exist from seasonal changes in thinning rate; \( p_{\text{eff}} \) is maximum in winter when thinning rate is minimum. During winter and early spring, when surface melting stops and precipitation is delivered as snow, effective pressure is relatively steady. During late spring and summer, rain and surface melt increase the variability of the basal water flux, thus
the variability of the effective pressure. Additionally, the large summertime basal water flux may cause frequent reorganizations of the subglacial hydraulic system in the dynamic terminus environment, and therefore increase the variability of effective pressure.

The length of LeConte Glacier varies seasonally, with maximum length spring, and minimum length in late fall or early winter. Assuming a constant annual near-terminus surface velocity (Chapter 2), there must be a change in the calving rate and glacier length by Equation (7). The changes may originate from the low and variable effective pressures that dominate upon the initiation of surface melting in spring. These changes may increase the calving rate, and decrease the length of the glacier. During summer, submarine melting also increases with rising ocean temperatures (Motyka and Hunter, unpublished data, 2000), and must contribute significantly to the late summer increase in calving rate.

**Unusual year**

Throughout its retreat, Columbia Glacier exhibited seasonal variations in speed, longitudinal stretching and terminus position, but for one year (the ‘unusual year’) during the early stage of its retreat, these cycles were severely damped or absent (Venteris and others, 1997). A distinct increase in these cycles followed, with seasonal variations now superimposed on a long-term increasing trend. Our 1999 study at LeConte Glacier could possibly be an ‘unusual year’ such as this one described for Columbia Glacier. An unusual year agrees with the steady speed over seasonal time scales (Chapter 2), as well a small change in length. However, the stretching rates at LeConte are very high compared with the stretching rates during the ‘unusual year’ at Columbia Glacier, which were the lowest documented for the interval spanning the first 5 years of retreat.
Conclusions

In this chapter, we have investigated the possibilities that ice flow to the terminus and/or short-lived buoyancy perturbations may control the frequency and timing of large calving events at LeConte Glacier.

Over time scales shorter than seasons, calving is more dependent on buoyancy forces than changes in the near-terminus ice velocity. This is supported by the lack of correlation between ice velocity and the occurrence of calving events. However, we did find that weekly averages of $U_i$ and $Q_c$ were both essentially constant over the study. Additionally, there were no observed changes in ice flow as a result of massive calving.

Instead, we observed a correlation between the floatation level of the glacier and the occurrence of calving events. First, a nearly floating terminus, provided by a deep water termination appears to be critical for rapid calving. This is indicated by the observation that there are no floating temperate tidewater glaciers. Terminal ice cliff heights for different tidewater glaciers appear to be nearly uniform regardless of water depth at the terminus.

Our observations indicate that it is perturbations about this state of near floatation that cause large calving events, with consequent increases in calving flux. These perturbations may be caused by various changes in glacier geometry, tidal amplitude, basal water pressure and storage, and possibly other factors.

On LeConte Glacier, a two to three day periodicity between large events suggests that ice thins to a critical level for calving. The duration between events may be influenced by other buoyancy factors, or possibly by flexure. Flexure of the nearly floating portion of the glacier may also be a significant perturbation leading to calving. Our observations show that a majority of large calving events occur during significant surface elevation drops, or
immediately after. This may be a result of transverse fracture propagation associated with the forward bending during the abrupt changes in surface elevation.

These triggering mechanisms all influence the magnitude and variability of the effective pressure beneath the terminus. Thus, if it were possible to measure variations in effective pressure there, a stronger correlation with the occurrence of calving events may result.
References


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Figures
Appendices
Appendix I. List of symbols

The following table presents a list of symbols used in the text and references the first use in the text.

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<thead>
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<th>Symbol</th>
<th>Description</th>
<th>First appearance</th>
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<tr>
<td>( \xi )</td>
<td>Longitudinal coordinate (km)</td>
<td>Fig. 1</td>
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<tr>
<td>( \tau_b )</td>
<td>Basal shear stress</td>
<td>Text p. 22</td>
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<tr>
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<td>Effective ice thickness</td>
<td>Equation (10)</td>
</tr>
<tr>
<td>( x )</td>
<td>Transverse position</td>
<td>Equation (10)</td>
</tr>
<tr>
<td>( W )</td>
<td>Glacier width</td>
<td>Equation (10)</td>
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<td>( q_c )</td>
<td>Calving flux over 50 m bands</td>
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<tr>
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<td>Definition</td>
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<tr>
<td>$\rho_w$</td>
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<td>$\mu$</td>
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Appendix II. Diurnal forcing- tests with synthetic data

We have developed a new function, (Equation 6) the admittance transfer function, for separating our high frequency ice velocity signal into two components. One component represents velocity variations forced by the tide, while the second component represents the portion of the signal driven by melt. To test the performance of the function, before applying it to our data, we performed tests on synthetic data.

A harmonic analysis of B* gave $U_{pred}$, the best fir predicted time series of the velocity for marker B*. We chose B* because of its strong tidal nature; this allowed us to easily determine the success of Equation 6 at removing the tidal portion of this signal. The predicted curve, $U_{pred}$, was then approximated with Equation (6). Figure A2.1 shows a good match between the two curves, except during times when $U_{pred}$ is a poor approximation of the observations, such as during data gaps and heavy rain (Figure 13: days 140-144). At these times, when $U_{pred}$ fails to approximate the observations, the admittance transfer function also fails. Nevertheless, the correlation coefficient (0.87) remains high, showing that the admittance transfer function is able to extract a tidal signal.

Next, we added a synthetic melt forcing term into the $U_{pred}$ time series. We used the $S_1$ tidal species with a predetermined phase angle (peak at 13:30) to model melt. This species has a period of 24.0 hours, so it is temporally non-migratory in nature, and somewhat resembles the smoothed ablation rate signal. The synthetic melt signal then has the form

$$\alpha_{M_2} \cdot 0.5A_{M_2}^{\text{tid}} \cos(\sigma_{M_2} \cdot t - \varphi)$$

(approximately 0.25 m d$^{-1}$ at the $S_1$ frequency). Adding this component perturbs the semi-diurnal nature of the velocity signal ($U_{pred}$), by strongly increasing the amplitude of one semi-diurnal peak and weakly decreasing the other (Fig. A2.2).
Extraction of the synthetic melt term ($S_1$) using the admittance transfer function was not perfect, as the admittance transfer function is only an approximation of the total tidally forced velocity. The largest errors occur at times when $U_{pred}$ is a poor approximation of the observations. It does, however, provide a satisfactory way for removing most of the tidally forced, semi-diurnal variations (Fig. A2.2). The residual, which should approximate the melt input, contained some semi-diurnal component, especially during the days of poor $U_{pred}$ fit, but Harmonic analysis of the residual gave the correct $S_1$ phase. Unfortunately, due to the modulating amplitude of this residual, the amplitude was not as well determined (Fig. A2.2). To improve the result, we applied harmonic analysis using both $S_1$ and $K_1$, allowing us to better match the modulating amplitudes in the admittance function residual. Then we were able to extract the correct $S_1$ amplitude and phase information from the total residual. Here, the input amplitude was 0.25 m and the input phase was $-149^\circ$. The $S_1$ residual had an amplitude of 0.24 m and a phase of $-150^\circ$. Thus, we feel that the function provides successful separation of a synthetic melt signal from a predominantly tidal signal; therefore it was applied to the motion data.
Appendix III. Photogrammetry

This appendix discusses the methods employed to obtain a time series of physical coordinates for the terminus of LeConte Glacier. While previous time-lapse photography analyses have used light table projection methods to digitize points of interest, we describe a new method that uses scanned copies of the negatives and computer software to quantify points of interest. Our method can provide similar or greater accuracy than the conventional method, especially if control points are known in each frame. It also incorporates low quality negatives, which were previously omitted, since enhancement of images acquired in poor weather is possible in phot editing software.

Each usable negative must be scanned at high resolution (~3072 x 2048 pixels). For maximum accuracy, the images must be scanned such that: 1.) the negative boundary is visible, or, 2.) at least two known control points are visible in the image. Satisfying one of these two criteria allows the screen coordinate in each image to be translated so that each image in the set has a similar reference frame. Unfortunately, our images contained no known control points, and scanning was performed in such a fashion that the negative boundary was excluded. This made an exact determination of the center (principal point) of each image impossible, and gave the effect of an unstable camera. However, due to the short focal length lens used in the main camera, the camera housing framed each image, allowing for a first order registration of the photo set (see Fig. 6).

After enhancing the contrast and brightness of any marginal images, each image was cropped to the window of the camera housing. To do this, we applied a color mask to each image using Corel Photo-Paint©. This aligned the center point of each image, with the
accuracy related to the blurriness of the camera housing window (dependent on weather conditions). This process did not alter the proportions of the image, and yielded a positional uncertainty of 3.9 m (50 mm lens) for a particle in the far field, and 2.1 m at the approximate glacier centerline. The images from the 100 mm camera were not framed by the inside of the box, resulting in a larger positional uncertainty of approximately 7.6 m in the far field and 4.2 m at the centerline.

The images were then organized in time sequential order (‘stacked’) using the image processing software “Scion Images” (http://rsb.info.nih.gov/nih-image/index.html) developed by the National Institute of Health. In each frame, the terminus position was digitized in a coordinate system \((\lambda, \mu)\) centered in the middle of the working window on the screen (screen frame; Fig. A3.1). Digitizing was performed in such a manner that the terminus outline is a true function of \(\mu\), allowing cubic spline interpolation at fixed intervals after the following coordinate transformation.

The exact position of any point may determined only by using two cameras in different locations. However, estimated coordinates may be obtained by intersecting the direction ray between the camera and any point, \(p\), with a known plane (Klimmel and Rasmussen, 1986). We estimated glacier topography with this plane using helicopter-borne GPS survey data, and marker A*'s elevation when it calved from the glacier. Although this gave a bimodal suggestion of the height of the terminus, we chose a plane 60 m above sea level for intersection with the direction ray array in each frame. As presented in Klimmel and Rasmussen (1986), the transformation of screen plane coordinates \((\lambda, \mu)\) to physical coordinates \((x', y', z')\) (Fig. A3.1) is accomplished by first applying three rotations about the pointing angles of the camera’s optic axis \((\theta, \phi, \psi)\), to obtain the direction ray \((X, Y, Z)\).
\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
x_o \\
y_o \\
z_o
\end{pmatrix} + \frac{\lambda_i}{\kappa} \begin{pmatrix}
\sin \theta \cos \psi - \cos \theta \sin \phi \sin \psi \\
- \cos \theta \cos \psi - \sin \theta \sin \phi \sin \psi \\
\cos \phi \sin \psi
\end{pmatrix} + \frac{\mu_i}{\kappa} \begin{pmatrix}
- \sin \theta \sin \psi - \cos \theta \sin \phi \cos \psi \\
\cos \theta \sin \psi - \sin \theta \sin \phi \cos \psi \\
\cos \phi \cos \psi
\end{pmatrix}
\]

where

\[
\begin{pmatrix}
x_o \\
y_o \\
z_o
\end{pmatrix} = \begin{pmatrix}
\cos \theta \cos \phi \\
\sin \theta \cos \phi \\
\sin \phi
\end{pmatrix}
\]

Here, \( \phi \) is the angle above the horizon, \( \psi \) is the transverse rotation from horizontal and \( \kappa \) is the focal length of the lens multiplied by the enlargement factor of the image from the original negative size (for the 50 mm lens \( \sigma = 7.38 \times 24 \text{ mm} \)). We simplify the equations of Krimmel and Rasmussen (1986) by setting the azimuth angle, \( \theta \), to 90°, such that the y axis is parallel to the camera’s optic axis.

The direction rays are then intersected with the plane representing the glacier.

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} = \frac{\gamma}{Z} \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\]

where \( \gamma \) is equal to the height above sea level, in this case 343 m. The results of the transformation are strongly dependent on \( \phi \), which was measured in the field at low levels of accuracy. A 1° change in \( \phi \) results in a 2 m change in \( x' \) when close to the measured dip angle, but may have much larger effect if \( \phi \) is small.

To constrain \( \phi \), we used a foot to mile map of the terminus region (Bowen, unpublished) to locate both the position of the time-lapse camera and a known rock outcrop along
the $y'$ axis. Then, by setting $\psi = 0$ and inverting the $y'$ component of Equation (A3.2) we solved for $\phi$:

$$
\phi = \tan^{-1} \left( \frac{y - \left( \frac{y' \mu}{K} \right)}{y' + \left( \frac{y' \mu}{K} \right)} \right)
$$

(A2.3)

and kept it constant in all subsequent images.

After obtaining physical coordinates ($x'$, $y'$, $z'$) for the position of terminus in the camera coordinate system, we sampled these curves every 5 m in $y'$ using cubic spline interpolation. This allowed successive frames to be differenced, yielding a change in area between frames. The local coordinates were also transformed to a UTM reference grid and plotted against GPS terminus surveys, showing the relative success of the transformation given the complications in the digitizing process (Fig. 2b).