# Continuum mechanics exercises

You're not expected to solve all problems, unless you're already well-versed in these methods. Start with the first part of Problem 1 and then pick an interesting looking problem from the remaining list.

## Problem 1: Practice with notation

- Write out the following expressions: $u_{ii}$, $\partial_i u_i$, $(\partial_i g)v_i$
- For the following quantities, state whether they are a scalar, a vector, or a second order tensor: $\partial_i T$, $\partial_j T$, $v_{ii}$, $\partial_j v_i$, $(\partial_j v_i)v_j$.
- Write out the expression $(\partial_j v_i)v_j$ that occurs in the momentum balance
- Check expression 2.12 by writing it out in component form

## Problem 2: Tensors

To solve this problem you need to be familiar with finding eigenvectors of a 2D matrix.
Angular momentum conservation implies that the stress tensor is symmetric, and that means that it can be diagonalized. That is, there is an orthogonal transformation (a rotation of the coordinate system) that makes the tensor diagonal. The values in the diagonal are the eigenvalues (principal stresses), and the directions in which they act are given by the eigenvectors.
The deformation rate field near the margin of a glacier can be approximated by 2D simple shear:

$$\mathbf{t} = \begin{pmatrix} 0 & \dot{\varepsilon} \\ \dot{\varepsilon} & 0 \end{pmatrix} \tag{1}$$

where $\dot{\varepsilon} = \frac{1}{2}du/dx$, with $u$ being the velocity component along the flow direction $x$.
Find the principal stresses and the directions and state what this means for crevasse orientation, by realizing that crevasses open perpendicular to the direction of maximum extension.

## Problem 3: Electrically charged ice?

Assume that ice was electrically charged with a charge density $\rho_c$. Use the general balance law (eqn. 2.8) to write down an equation that expresses the conservation of electrical charge.
*Hint:* Charge can be described by a charge density (just like mass). It is a conserved quantity, so the supply is 0. Finally, there is flux of charge, given by the electric current.

## Problem 4: Accelerating ice flow

Estimate the relative size of the acceleration term in the momentum balance and state whether the Stokes approximation appears to be suitable for glaciological purposes.

## Problem 5: Flow law

Show that the tensor form of the flow law reduces to (2.35) for simple shear.

## Problem 6: Navier-Stokes equation

Use the momentum balance derived in the manuscript and replace the stresses with strain rates to write down the Navier-Stokes equation for slow non-linear fluids.
Reduce this equation to a simpler one for flow through a glacier cross-section. Assume that there is only one non-zero velocity component ($\mathbf{v} = (u, 0, 0)^T$), and all out-of-plane gradients are zero ($\frac{\partial}{\partial x} = 0$).

## Problem 7: Energy conservation

Assume that the lower 5 km of Jakobshavn Isbræ has a constant cross-sectional area of 5 km width and 1 km depth, a surface slope of 2°, and flows at an average speed of 20 m/d. How much potential energy is dissipated in this process. At what rate would this warm this volume of ice? If all the energy went into melting instead, how much ice would melt?