Elevation and Volume Change:

Aircraft- and satellite-mounted sensors generate large data sets that contain enormous amounts of information about glacier systems and their changes. To determine quantitative characteristics of a glacier system requires special processing of these datasets. One parameter that is highly valuable for monitoring glacier changes is elevation. Changes in glacier elevations can tell us a lot about a glaciers response to climate perturbations and deviation in glacier flow from steady state conditions. Elevation differencing provides one of the most spatially complete and detailed observations of glacier change.

Elevation datasets provide surface heights referenced to a vertical datum. The reference datum is either an idealized mathematical representation of the Earth’s surface (Ellipsoid) or a surface of equal potential energy (Geoid: the surface the Earth would have if it was liquid). The Earth’s geoid changes over time due to Glacio Isostatic Adjustment (GIA), changes in sea level, changes in glacier and ice sheet mass, and other global mass redistribution (even the construction of dams!). In addition, many older elevation maps and Digital Elevation Models (DEMs) are referenced to local datums. Before elevation differencing can be done, all elevations must be referenced to the same datum. Most conversions can be done using standard GIS software or with the Geospatial Data Abstraction Library (GDAL). In certain cases lookup tables must be used.

Figure 1: Illustration showing Geoid and Ellipsoid referenced heights.
The Earth's topography can be measured using several different sensors and techniques. Three common approaches used in glaciology are:

1. Laser altimetry (e.g. NASA ATM, NASA IceBridge, ICESat)
2. Stereoscopic imagery (e.g. SPOT, ASTER, airborne surveys)
3. Radar (e.g. STRM, CryoSat-2)

Each approach has its advantages and disadvantage. Laser altimetry is very precise but often has poor spatial coverage and can suffer from cross-track slope errors if not properly corrected. Stereoscopic imagery has very good spatial coverage but often has poor temporal sampling and requires surface contrast, something that is lacking over the interior ice caps and ice sheets. Radar provides the best spatial coverage but can suffer from elevation errors due to the penetration of the radar signal into low-density snow and firn. While all products have their limitations, with proper correction, very accurate elevation and volume change estimates can be made.

ICESat provides elevations with decimeter accuracies. Optically, clouds are nearly identical to snow and can reflect the ICESat laser pulse giving an erroneously high elevation. In addition, ICESat elevation sampling is near repeat-track (not exact) and therefore tracks cannot be directly differenced, as errors due to cross-track slope are often larger than the detectable elevation change. One way to overcome this limitation is to difference crossover points between ascending and descending orbits (Fig2. a). This has worked well for the Greenland and Antarctic Ice Sheets (Brenner et al., 2007) but provides too few points for monitoring elevations over glaciers and ice caps. An alternative to this approach is to employ a least-squares fitting of rigid planes to segments of repeat-track data assuming a constant elevation change rate, thus correcting for cross-track slope (Fig. 2.c: Moholdt et al., 2010). This approach also has the added benefit of producing mean elevation planes to which an elevation deviation threshold can be applied to filter out erroneous cloud returns. Moholdt et al., 2010 provide good detailed reviews of ICESat processing methods as applied over Svalbard glaciers.
Elevations are derived from stereoscopic images (image of the same object but from different locations) by measuring the parallax displacement, that is the relative displacement of a fixed object relative to other fixed objects with a change in location of the observer. Observation of parallax displacement requires image contrast and therefore DEMs generated from stereoscopic images often have many interpolated points over low contrast snow and ice and can contain large blunders that must be identified before elevation differencing can take place. In addition, elevations derived from stereoscopic images often have very good relative elevation control but can have poor absolute control and therefore benefit from the use of ground control points. In some cases, such as SPOT SPIRIT DEMs (Korona et al., 2009), DEMs are generated using purely automated methods and require global vertical corrections before they can be used for elevation differencing.

Even if elevation products are referenced to the same vertical datum, there can still be georeferencing errors that result in horizontal offsets between products. This requires co-registration. Two elevation products of the same terrain surface that are mis-aligned will experience a characteristic relationship between elevation differences and the direction of the terrain (aspect) that is precisely related to the horizontal shift vector (Fig. 3). Using this relationship, the horizontal offset between two corresponding products can be determined and corrected for. Nuth and Kaab, 2011 provide a detailed step-by-step guide for co-registering DEMs using this approach. Note that corrections are done over ice-free terrain.
In addition to horizontal and globally applied vertical correction, there may also be spatially correlated biases that can propagate into the estimation of glacier elevation changes. These spatially correlated biases can result from spatially varying rates of GIA, geoid conversion errors, and control point errors. Elevation differences over ice-free ground should be assessed to identify spatially correlated biases, which can then be corrected using a fitted surface (Gardner et al., 2012).

**Converting Elevation Change to Volume Change**

Glacier elevation changes are most often spatially discontinuous and must be extrapolated over the glacier surface to determine total changes in glacier volume. The surface mass budget of nearly all glacier systems is strongly dependent on elevation. This relationship can be exploited to extrapolate sparse elevations over a glacier surface. This requires detailed knowledge of the total glacier area-per-elevation interval (hypsometry), which can be determined using glacier outlines and a DEM. The mean or median elevation change for each elevation interval (bin) is determined or parameterized using a n-order polynomial fit that is then multiplied by the hypsometry to determine change in glacier volume. This approach has been shown to work with sufficient RANDOM sampling (Gardner et al., 2012; Gardner et al., 2011; Moholdt et al., 2012; Moholdt et al., 2010; Nuth et al., 2010) but can have larger biases with non-RANDOM (centerlines) and/or insufficient sampling (Berthier et al., 2010; Gardner et al., 2012). Careful attention should be given to use of elevation change bin means (capture anomalous elevation changes) vs. medians (less sensitive to outliers).
Estimating the uncertainty of the volume change estimate can be more difficult than estimating the volume change itself. There are many sources of uncertainty:

1. Uncertainties in the measurement of elevation change:

Uncertainties in elevations can be determined through examination of elevation differences over ice-free terrain ($E$). Most often, the standard difference between elevation products is assumed to be the uncertainty in the elevation differencing over ice. To determine the uncertainty of the volume change estimate, the correlation length of the error needs to be determined. This can be estimated using semiveriogram analysis. The simplest approach is to identify a correlation length through visual inspection of the semiveriogram (transition to zero slope on variance vs. distance plot). The correlation length is then used to determine the number ($N$) of independent observations. The measurement uncertainty in the volume change ($\sigma$) is then estimated as the root-sum-of-squares of the independent observations:

$$\sigma = \frac{E}{\sqrt{N}}$$
More sophisticated approaches to determine the uncertainty of the area averaged elevation change are discussed in detail by Rolstad et al., 2009.

2. Uncertainties in the extrapolation of elevation change to determine volume changes:

This can be estimated by comparing independent volume changes estimates generated using different datasets or by extrapolating subsets of elevation change datasets and comparing resulting volume change.

3. Glacier area change between elevation measurements.

This can be estimated through examination of the sensitivity of the volume changes to changes in the hypsometric areas at lower elevation bins. Even better would be to have glacier outlines for each of the elevation acquisition dates, but this is usually not practical.

4. Elevation changes from GIA (only maters if not vertically co-registered over land)

Can be estimated from measured or modeled GIA.

**Volume Change to Mass Change**

Converting volume changes to mass changes is complicated by changes in the near-surface (top 5-100 meters) density profile. For dry snow-zone regions, if the accumulation and temperature are constant in time then the density profile will also constant (Sorge’s law). In such cases measured elevation changes can be directly converted to mass changes by multiply the elevation change by the density of ice (917 kg m\(^{-3}\)). This is, however, rarely the case (Fig. 5) and the density profile must be either measured or modeled before meaningful mass change estimates can be made. Over the ice sheets, where the dry-snow zone covers large areas, relatively small uncertainties in the density profile result in large uncertainties in the overall mass change estimate. Glaciers have relatively smaller areas of deep firn-pack than ice sheets but uncertainties can still be large. There are many empirical and physically based models of snow densification that are reviewed by Arthern et al., 2010. However, these models do not account for melt processes that can lead to rapid compaction of snow and firn. Modeling such processes requires physically based snow/firn/ice column models that are forced at their surface with climate reanalysis, weather station data, or regional climate model output (Ligtenberg et al., 2011).
Estimating glacier mass changes using GRACE

An excellent introduction to measuring mass changes (glaciers included) using time variable gravity (GRACE) has been written by John Wahr, University of Colorado Boulder, and can be found at:

www.gps.caltech.edu/classes/ge167/file/Wahr_Gravity_treatise.pdf
References: