On Notation

▶ (hopefully) consistent with Continuum Mechanics (Truffer)
▶ with lots of input from Luethi & Funk: Physics of Glaciers I lecture at ETH
▶ notation following Greve & Blatter: Dynamics of Ice Sheets and Ice Sheets

Types of Glaciers

cold glacier
ice below pressure melting point, no liquid water

temperate glacier
ice at pressure melting point, contains liquid water in the ice matrix

polythermal glacier
cold and temperate parts

Why we care

The knowledge of the distribution of temperature in glaciers and ice sheets is of high practical interest
▶ A temperature profile from a cold glacier contains information on past climate conditions.
▶ Ice deformation is strongly dependent on temperature (temperature dependence of the rate factor $A$ in Glen’s flow law).
▶ The routing of meltwater through a glacier is affected by ice temperature. Cold ice is essentially impermeable, except for discrete cracks and channels.
▶ If the temperature at the ice-bed contact is at the pressure melting temperature the glacier can slide over the base.
▶ Wave velocities of radio and seismic signals are temperature dependent. This affects the interpretation of ice depth soundings.
Energy balance: depicted

- Surface energy balance
- Frictional heating
- Latent heat sources/sinks
- Strain heating
- Geothermal heat (firn + near surface layer)

Energy balance: equation

\[ \rho \left( \frac{\partial U}{\partial t} + \mathbf{v} \cdot \nabla U \right) = -\nabla \cdot \mathbf{q} + Q \]

\( \rho \) ice density
\( U \) internal energy
\( \mathbf{v} \) velocity
\( \mathbf{q} \) heat flux
\( Q \) dissipation power (strain heating)

Noteworthy

- Strictly speaking, internal energy is not a conserved quantity
- Only the sum of internal energy and kinetic energy is a conserved quantity

Case study: Colle Gnifetti

- Uppermost part of Grenzgletscher, Monte Rosa, at an altitude of \( \sim 4500 \text{ m} \)
- Mean annual air temperature of \( \sim -13 \text{ °C} \)
- Amplitude of \( \sim 7 \text{ °C} \)

- How does a temperature in the first 15 look like?

Cold ice

Temperature equation

- Ice is cold if a change in heat content leads to a change in temperature alone
- Independent variable: temperature \( T = c(T)^{-1} u \)

\[ \rho c(T) \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -\nabla \cdot \mathbf{q} + Q \]

Fourier-type sensible heat flux

\[ \mathbf{q} = \mathbf{q}_s = -k(T) \nabla T \]

- \( c(T) \) heat capacity
- \( k(T) \) thermal conductivity
### Thermal properties

- **Heat capacity** is a monotonically-increasing function of temperature.

- **Thermal conductivity** is a monotonically-decreasing function of temperature.

### Flow law

Viscosity $\eta$ is a function of effective strain rate $d_e$ and temperature $T$:

$$\eta = \eta(T, d_e) = 1/2B(T)d_e^{(1-n)/n}$$

where $B = A(T)^{-1/n}$ depends exponentially on $T$.

### Ice temperatures close to the glacier surface

#### Assumptions
- Only the top-most 15 m experience seasonal changes.
- Heat diffusion is dominant.

We then get

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial h^2}$$

where $h$ is depth below the surface, and $\kappa = k/(\rho c)$ is the thermal diffusivity of ice.

#### Boundary Conditions

- $T(0, t) = T_0 + \Delta T_0 \cdot \sin(\omega t)$,
- $T(\infty, t) = T_0$.

- $T_0$ mean surface temperature
- $\Delta T_0$ amplitude
- $2\pi/\omega$ frequency
Ice temperatures close to the glacier surface

Analytical Solution

\[ T(h, t) = T_0 + \Delta T_0 \exp \left( -h \sqrt{\frac{\omega}{2\kappa}} \right) \sin \left( \omega t - h \sqrt{\frac{\omega}{2\kappa}} \varphi(h) \right). \]

\( \Delta T(h) \) amplitude variation with depth

Case study: Colle Gnifetti

Assumptions

▶ only vertical advection and diffusion

We then get

\[ \kappa \frac{\partial^2 T}{\partial z^2} = w(z) \frac{\partial T}{\partial z} \]

where \( w \) is the vertical velocity

Analytical solution

▶ can be obtained
Cold Glaciers

- Dry Valleys, Antarctica
- (very) high altitudes at lower latitudes

Temperate ice

Water content equation

- Ice is temperate if a change in heat content leads to a change in water content alone
- independent variable: water content (aka moisture content, liquid water fraction) \( \omega = L^{-1} u \)

\[
\rho L \left( \frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega \right) = -\nabla \cdot \mathbf{q} + Q
\]

⇒ in temperate ice, water content plays the role of temperature

Flow law

Flow Law

Viscosity \( \eta \) is a function of effective strain rate \( d_e \) and water content \( \omega \)

\[
\eta = \eta(\omega, d_e) = 1/2B(\omega)d_e^{(1-n)/n}
\]

where \( B \) depends linearly on \( \omega \)

- but only very few studies (e.g. from Lliboutry and Duval)

Latent heat flux

\[
\mathbf{q} = \mathbf{q}_l = \begin{cases} 
\text{Fick-type} \\
\text{Darcy-type}
\end{cases}
\]

⇒ leads to different mixture theories (Class I, Class II, Class III)

Sources for liquid water in temperate Ice

1. water trapped in the ice as water-filled pores
2. water entering the glacier through cracks and crevasses at the ice surface in the ablation area
3. changes in the pressure melting point due to changes in lithostatic pressure
4. melting due energy dissipation by internal friction (strain heating)
Temperature and water content of temperate ice

Temperature

\[ T_m = T_{tp} - \gamma(p - p_{tp}), \]  

(1)

- \( T_{tp} = 273.16 \text{ K} \) triple point temperature of water
- \( p_{tp} = 611.73 \text{ Pa} \) triple point pressure of water
- Temperature follows the pressure field

Water content

- generally between 0 and 3%
- water contents up to 9% found

Temperate Glaciers

Temperate glaciers are widespread, e.g.:

- Alps, Andes, Alaska,
- Rocky Mountains, tropical glaciers, Himalaya

Polythermal glaciers

- contains both cold and temperate ice
- separated by the cold-temperate transition surface (CTS)
- CTS is an internal free surface of discontinuity where phase changes may occur
- polythermal glaciers, but not polythermal ice

Scandinavian-type thermal structure

- Scandinavia
- Svalbard
- Rocky Mountains
- Alaska (e.g. McCall Glacier, see exercise)
- Antarctic Peninsula
Scandinavian-type thermal structure

Why is the surface layer in the ablation area cold? Isn’t this counter-intuitive?

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Canadian-type thermal structure

- high Arctic latitudes in Canada
- Alaska
- both ice sheets Greenland and Antarctica

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Thermodynamics in ice sheet models

- only few glaciers are completely cold
- most ice sheet models are so-called *cold-ice method* models
- so far two polythermal ice sheet models

\[
\rho c(T) \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = \nabla \cdot k \nabla T + Q
\]

\[
\rho L \left( \frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega \right) = Q
\]

or

\[
\rho \left( \frac{\partial E}{\partial t} + \mathbf{v} \cdot \nabla E \right) = \nabla \cdot \nu \nabla E + Q
\]
Why polythermal is better: Antarctica

- conservation of energy
- more realistic basal melt rates
- more realistic ice streams

Why polythermal is better: Greenland

- conservation of energy
- more realistic basal melt rates
- more realistic ice streams

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