The Philosophy of Glacier Motion

McCarthy Summer School, June 2018

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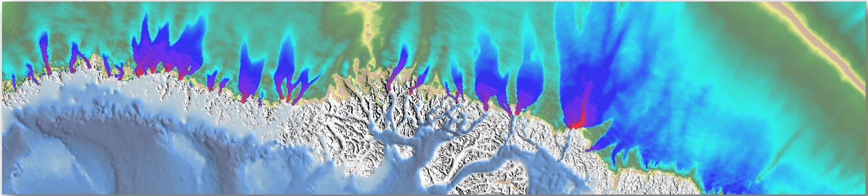
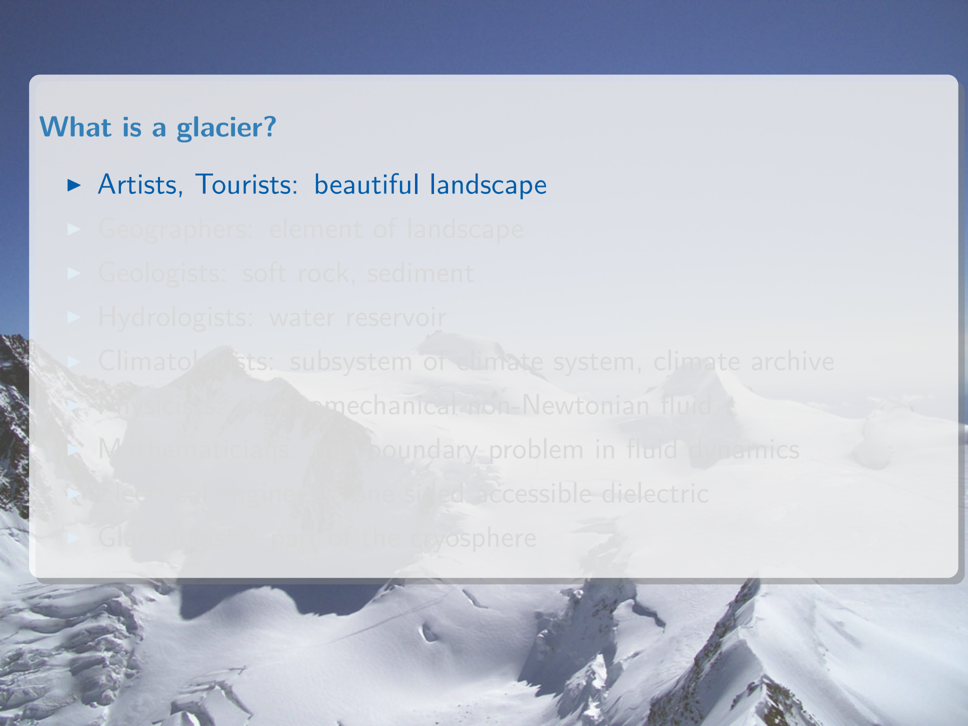
University of Alaska Fairbanks, USA



Overview

- selected examples of glacier flow, using concepts from Continuum Mechanics
- and some historical background



What is a glacier?

- Artists, Tourists: beautiful landscape
- Geographers: element of landscape
- Geologists: soft rock, sediment
- Hydrologists: water reservoir
- Climatologists: subsystem of climate system, climate archive
- Physicists: thermomechanical non-Newtonian fluid
- Mathematicians: free boundary problem in fluid dynamics
- Electrical engineers: one sided accessible dielectric
- Glaciologists: part of the cryosphere

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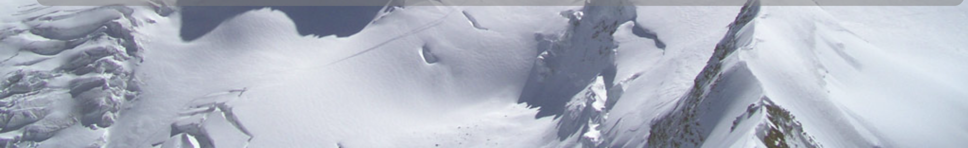

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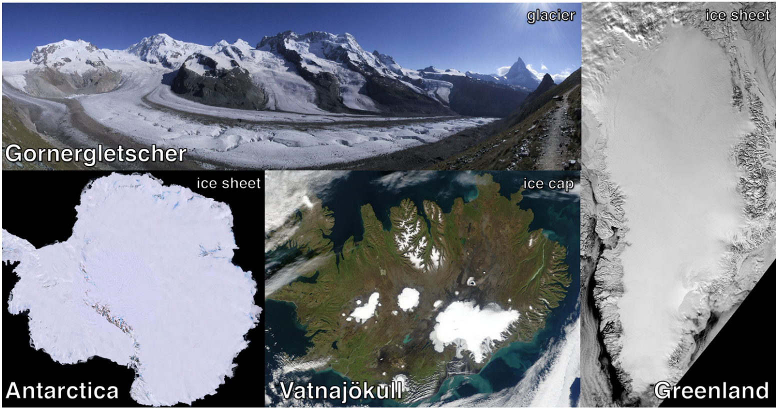
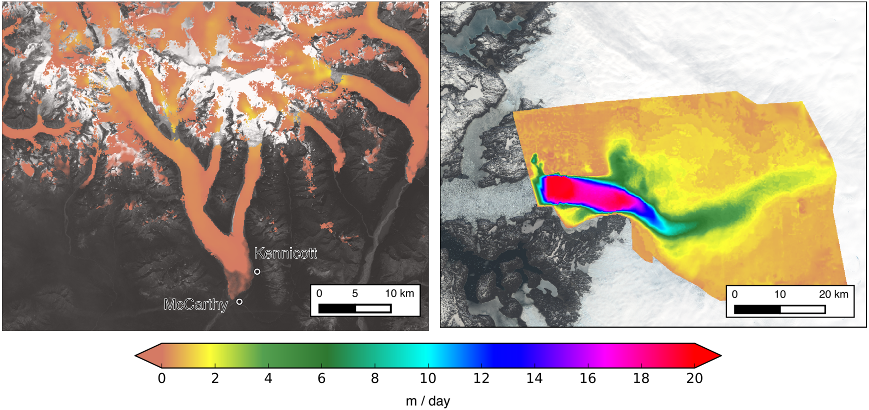


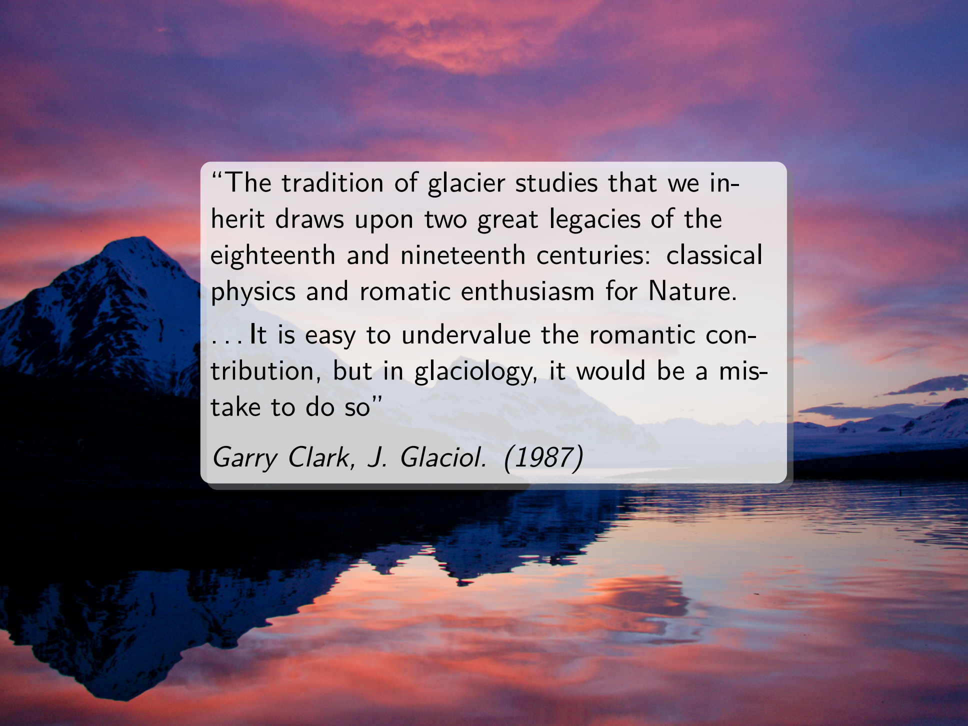
- Glaciers flow slowly under their own weight

Glacier flow

speeds by Evan Burgess (Wrangell) and Roman Motyka (Jakobshavn)



- observed speeds range from $20\,\mathrm{m\,a^{-1}}$ for a valley glacier to $15\,\mathrm{km\,a^{-1}}$

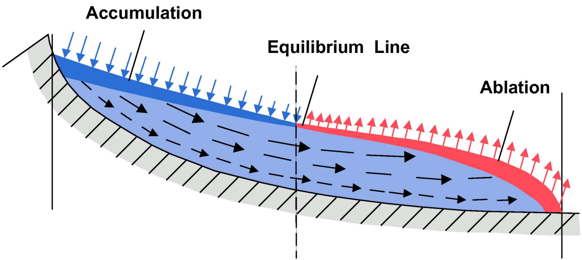


"The tradition of glacier studies that we inherit draws upon two great legacies of the eighteenth and nineteenth centuries: classical physics and romatic enthusiasm for Nature.

. . . It is easy to undervalue the romantic contribution, but in glaciology, it would be a mistake to do so"

Garry Clark, J. Glaciol. (1987)

How does a glacier move?



In a nut shell:

- Well-described by continuum mechanics (Martin's lecture)
- The ice can deform as a viscous fluid
- The ice can slide over its substrate

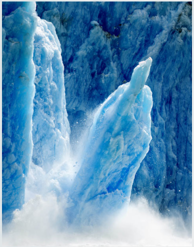
Water plays an important role

Truffer's Law

- whenever something interesting happens in a glacier, liquid water is involved

The role of liquid water

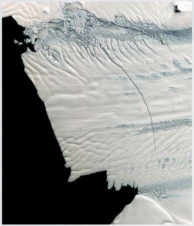
at the lateral margin



drives calving and frontal melt of lake and ocean terminating glaciers
lecture on tidewater glaciers and submarine melt by Martin Truffer

The role of liquid water

at the shelf base



attacks ice shelfs from below (sub-shelf basal melting)
lecture on tidewater glaciers and submarine melt by Martin Truffer

The role of liquid water

at glacier base

acts as a lubricant at the base (sliding)
lecture on subglacial hydrology by Matt Hoffman
surging glaciers

The role of liquid water

within temperate ice

softens the ice (decreases viscosity)
lecture on thermodynamics by Andy

Measurements & Observations

1779 Gravitation theory by de Saussure

H. B. de Saussure observes sliding

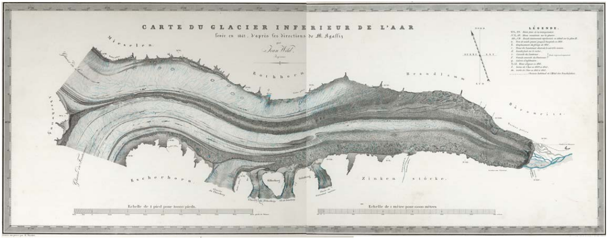
- "...the weight of the ice might be sufficient to urge it down the slope of the valley, if the sliding motion were aided by the water flowing at the bottom."

1827-1836 Hugi Block

J. Hugi observed that a boulder moved 1315 m downstream between 1827 and 1836

- we would interpret this as clear evidence of glacier flow
- but back then, some people argued that a boulder slides on the glacier surface, the glacier itself is motionless

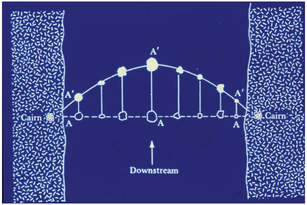
Measurements & Observations

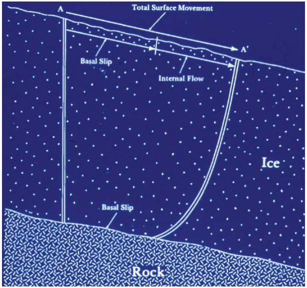


1840-1846 Dilatation theory by L. Agassiz

- glacier ice contains innumerable fissures and capillary tubes
- during the day, these tubes absorb the water
- and during the night, the water freezes
- this distension exerts a force and propels the glacier in the direction of least resistance

Measurements & Observations





1864-1930 Viscous flow theory by J. Forbes

- made his own observations on Mer de Glace, France
- glacier flows fastest in the center
- opposes Agassi's theory
- if the dilatation theory were true
- then flow would be greatest at sunset
- and near the glacier margins

Measurements & Observations

The Philosophy of Glacier Motion
Author(s): Wm. Luttrell Rogers
Source: *Journal of the American Geographical Society of New York*, Vol. 20 (1888), pp. 481-500+497-501
Published by: American Geographical Society
Stable URL: http://www.jstor.org/stable/196776
Accessed: 09-06-2018 23:48 UTC

Forces

- a force is a push or pull upon an object resulting from the object's interaction with another object
- whenever there is an interaction between two objects, there is a force upon each of the objects.

In other words

- a force is any influence that causes an object to undergo a change in speed, a change in direction, or a change in shape
- forces exist inside continuous bodies such as a glacier
- these forces can cause a glacier to deform

What is stress?

"Stress" is the force per unit area acting on a material

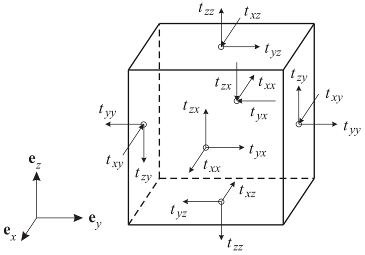
$$\sigma = \frac{F}{A}$$

Inside a glacier, stresses are due to

- weight of the overlying ice (overburden pressure)
- shape of the glacier surface (pressure gradients)

Types of stress

As a force per unit area, stress has a direction



Force can be directed normal to the area

- Result is pressure if the force is the same on all faces of a cube.
- Result is normal stress if forces are different on different faces

Force can be directed parallel to the area

- Result is shear stress

Pressure in a glacier

mass $m = \rho V$

- $\rho =$ ice density $\approx 900\,\mathrm{kg\,m^{-3}}$
- $V =$ Volume $=$ Area $\times$ depth $= A \cdot h$

So pressure p at depth h is

$$p = \frac{m \cdot g}{A} = \frac{\rho \cdot A \cdot h \cdot g}{A} = \rho g h + p_{\mathrm{air}}$$

How deep do we have to drill into a glacier before the ice pressure is 2 atmosphere?

Depth for 2 atm pressure?

$$h = \frac{p - p_{\mathsf{air}}}{\rho \cdot g} = ?$$

- So pressure rises by 1 atm for every x meters of depth in a glacier
- Does ice deform in response to this pressure?

Shear stress τ

Total stress t from ice column:

$$t = \frac{\rho V g}{A} = \rho\, g\, h$$

- How much of this weight will contribute to shear deformation?
- shear stress $\tau = \rho\, g\, h \sin\alpha$
- normal stress $\sigma = \rho\, g\, h \cos\alpha$

Shear stress in a glacier

Valley Glacier

- $h = 130\,\mathrm{m}$
- $\alpha = 5^\circ$

Ice Sheet

- $h = 1300\,\mathrm{m}$
- $\alpha = 0.5^\circ$

$$\tau = \rho\, g\, h \sin\alpha$$

Shear stress at the glacier base, τ_b, is $\approx 1\,\mathrm{bar}$, which is a typical value for basal shear stress under a glacier

Are glacier thickness and slope related?

Suppose a glacier becomes thicker or steeper due to mass imbalance:

- it flows faster
- it quickly reduces thickness h or slope α, until $\tau_b \approx 1\,\text{bar}$ again

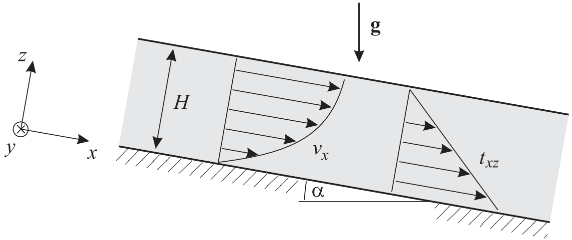
Can we then estimate glacier thickness ($z = h$) from its slope if we know $\tau_b \approx 1\,\text{bar}$?

$$\tau = \rho\, g\, h \sin\alpha \quad \Rightarrow \quad h \sim \frac{\tau_b}{\rho\, g\, \sin\alpha}$$

An ice sheet of infinite height?

- power-law stress-strain relationship $\Rightarrow$ ice softens rapidly as the shear stress exceeds 1 bar.
- ice flow also increases rapidly $\Rightarrow$ glacier expands and thins
- that 1 bar is a typical stress is a result of A and n

Parallel Sided Slab



Assumptions

- uniform in x- and y-direction $\Rightarrow \partial/\partial x = \partial/\partial y = 0$
- steady-state $\Rightarrow \partial/\partial t = 0$
- stress free surface $\Rightarrow \mathbf{T} \cdot \mathbf{n}|_{z=H} = 0$
- no-slip at the base $\mathbf{v}|_{z=b} = 0$

$$\dot{\varepsilon}_{xz} = A\tau^{n-1}\sigma_{xz}^{(d)}$$

Parallel Sided Slab

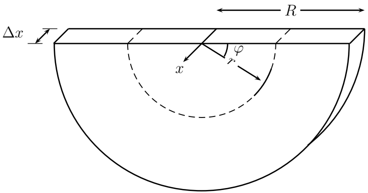
$$
\begin{aligned}
\frac{\partial v_x}{\partial z} &= 2\,A(T,p)\,\tau^{n-1}\tau \qquad \Rightarrow \\
v_x(z) &= \int_0^h 2\,A\,(\rho\, g\, \sin\alpha\, z)^n\; dz \\
&= \frac{2\,A}{n+1}\,(\rho\, g\, \sin\alpha)^n\, \left(h^{n+1} - (h-z)^{n+1}\right)
\end{aligned}
$$

See script p. 4–10 for a derivation

Noteworthy

- horizontal velocity grows with n-th power of surface slope
- horizontal velocity grows with n+1-th power of ice thickness
- this is an analytical solution, can be used for code verification (Ed's lecture)

Flow of a Glacier in a Semi-Circular Valley



Assumptions

- uniform in x- and φ-direction, steady-state
  $\Rightarrow \partial/\partial x = \partial/\partial\varphi = \partial/\partial t = 0$
- body force has to be balanced by tractions acting on the circumference in distance r from the center

$$\sigma_{rx}\pi r \Delta x = -\rho\, g\, \frac{\pi r^2}{2} \Delta x \sin\alpha$$

Flow of a Glacier in a Semi-Circular Valley

Similar to the parallel-sided slab, an analytical solution for the flow through a cylindrical channel can be obtained.

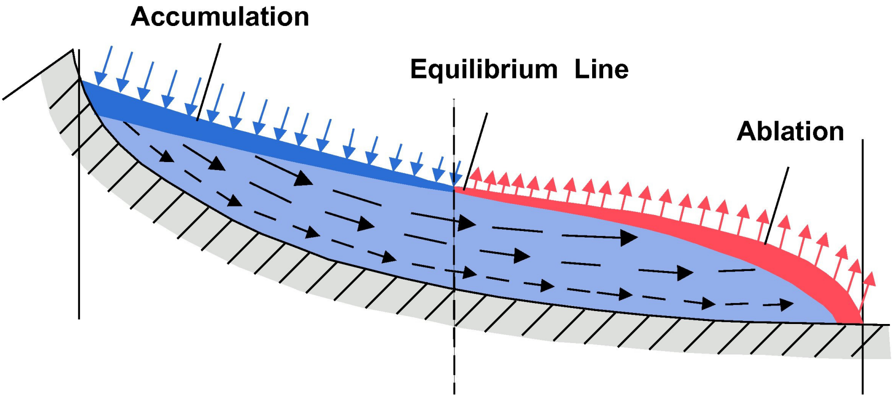
$$v_x(r) = v_x(0) - \frac{2A}{n+1} \left(\frac{1}{2}\rho\, g\, \sin\alpha\right)^n r^{n+1}$$

Noteworthy

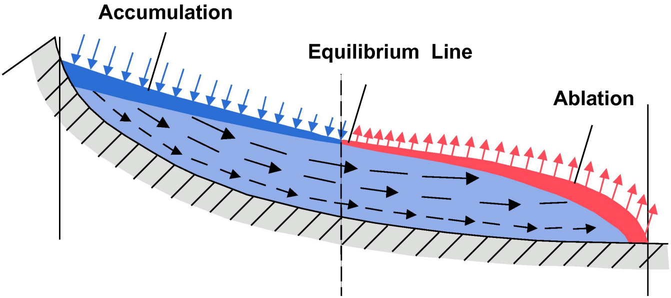
$$v_{x,\text{channel}} = \left(\frac{1}{2}\right)^n v_{x,\text{slab}}$$

- here, the radius R plays the role of the ice thickness H
- center line velocity is eight times slower than in an ice sheet of the same thickness (side wall drag)

Longitudinal Profile of a Valley Glacier



Evolution of the Glacier Surface



Derivation on p. 12–14. Assuming $\partial b/\partial t = 0$ and no basal melt:

$$\frac{\partial h}{\partial t} = \underbrace{-\nabla \cdot \mathbf{Q}}_{\text{dynamic changes}} \quad \underbrace{+B_{\text{clim}}}_{\text{climatic changes}}$$

Why ice sheet modeling is easy


G. G. Stokes

- composed of a single, largely homogeneous material
- flow governed by the Stokes equations known since the mid-19th century
- flows slowly: we can ignore turbulence, Coriolis and other inertial effects
- no density/salinity stratification
- most of what makes atmosphere and ocean flow interesting is missing

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On turbulence

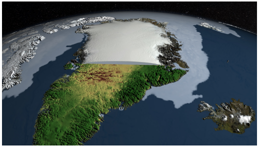
"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic." (O. Reynolds)

- so what makes the flow of slow, cold, laminar ice interesting?

Why ice sheet modeling is so hard

Boundary conditions

- seaward margin boundary condition
- basal boundary condition



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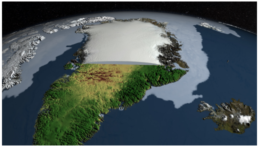

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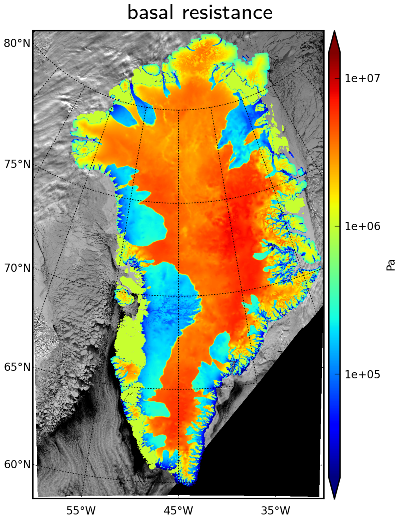
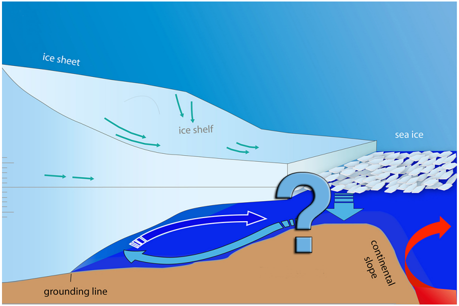
Challenge: basal boundary condition



- stresses vary by orders of magnitude
- basal hydrology, the glacier's "plumbing system" runs on a faster time scale than ice flow
- despite more than 5 decades of research, we only have crude parametrizations

$\Rightarrow$ Martin's inverse method lecture

Challenge: seaward margin boundary condition



- ocean circulation $\Rightarrow$ basal melt rates
- calving mechanism

$\Rightarrow$ Martin's tidewater glacier lecture

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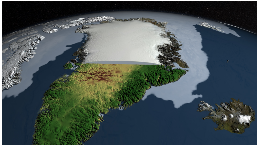

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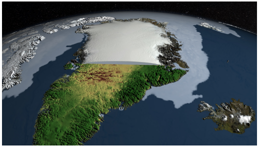

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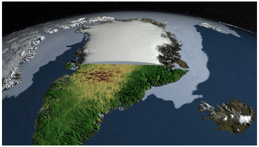

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1. Simplify the Stokes equations

Ice sheets are shallow

- below in red is a no-vertical-exaggeration cross section of Greenland at 71°
- green and blue: standard vertically-exaggerated cross section

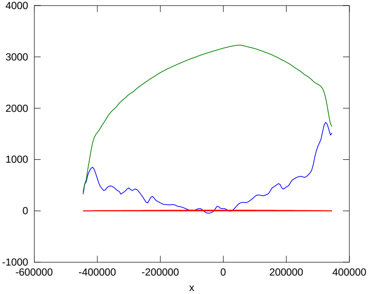


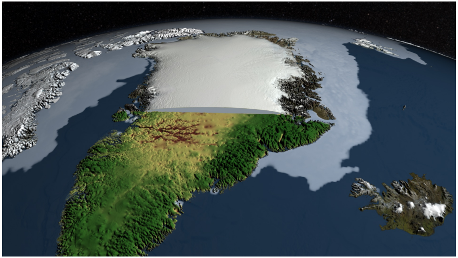
Figure by E. Bueler

2. High Performance Computing (HPC)

Exploit modern HPC through parallelism

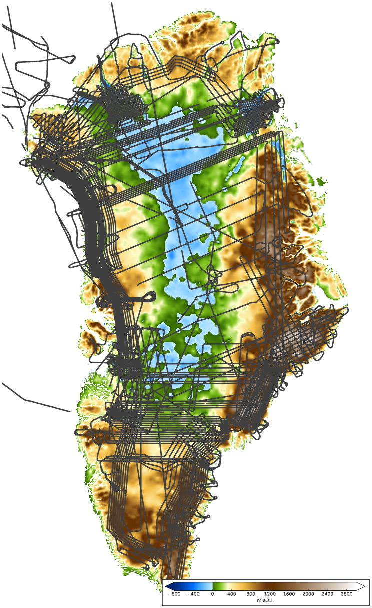
Challenge: ice thickness

- ice thickness is a leading order constraint on ice flow
- but expensive to measure
- NASA realized that collecting a lot more ice thickness measurements is crucial to make ice sheet models better
- ice thickness measurements using the CReSIS radar became an important part of their Operation IceBridge mission (2009–today)

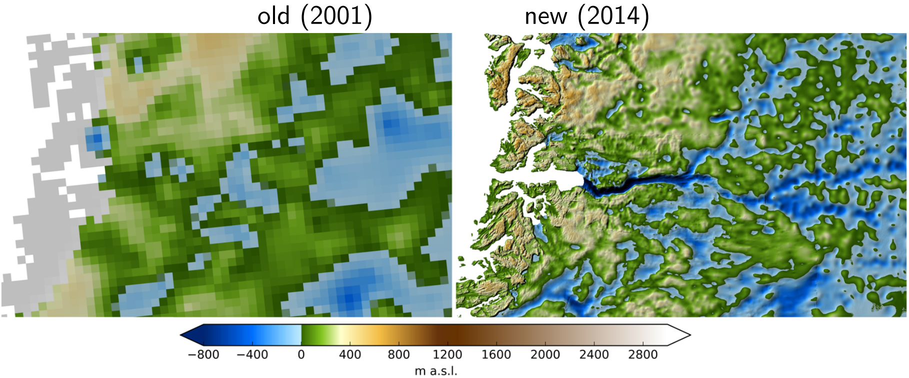


NASA Operation IceBridge

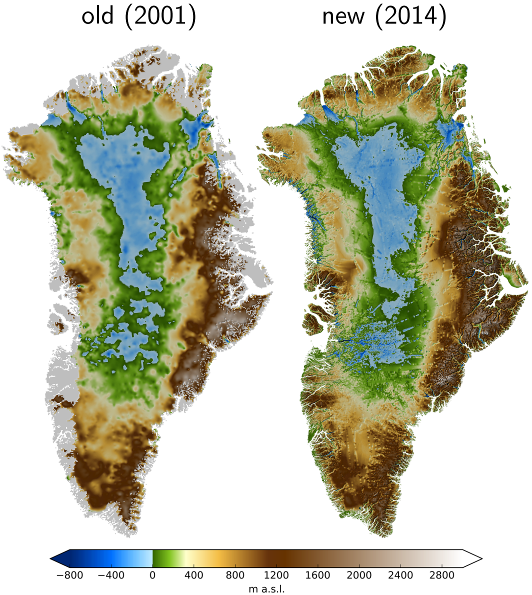
- additional flight lines since 2009



Zoom in to Jakobshavn Isbræ



NASA Operation IceBridge

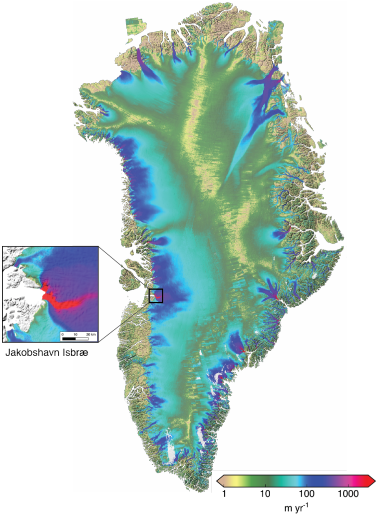


- from 5 km to 150 m horizontal grid resolution

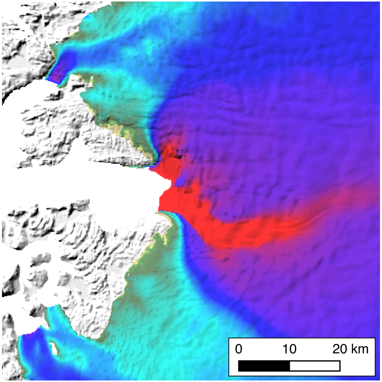
NASA Operation IceBridge

- do NASA's OIB million$ really make ice sheet models better?

Observed flow speeds

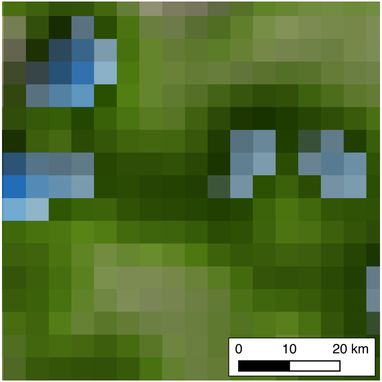


Jakobshavn Isbræ



Ice thickness and simulated flow speeds

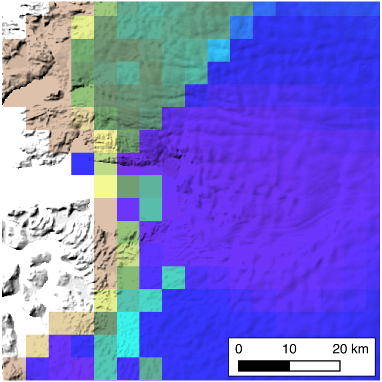
Jakobshavn Isbræ



5 km, old data set (2001)

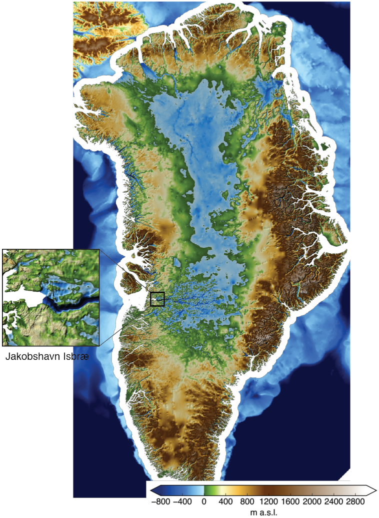
Ice thickness and simulated flow speeds

no fast flow



simulated surface speed

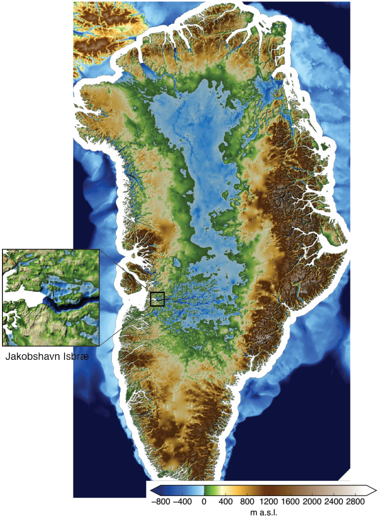
Ice thickness and simulated flow speeds



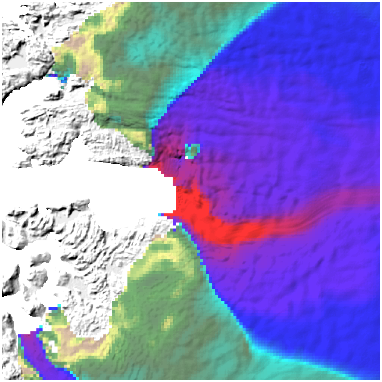
Jakobshavn Isbræ

600 m, new data set (2014)

Ice thickness and simulated flow speeds



fast flow appears



simulated surface speed

Can we capture the present-day flow field?