Observations

Theory

Modeling

Conduit in Kötlujökull (Näslund and Hassinen, 1996)

\[
\bar{v} = -\left( \frac{4 S^{1/2}}{f_r \rho_w \pi^{1/2}} \right)^{1/2} \frac{\partial \phi}{\partial s} \left| \frac{\partial \phi}{\partial s} \right|^{-1/2}
\]
Governing equations for subglacial drainage

Continuity

\[ \frac{\partial h}{\partial t} + \nabla \cdot q = b \]

Source/sinks

\[ b = b_s + b_e + b_a + \frac{Q_G + Q_F}{\rho_i L} \]

Fluid potential

\[ \phi = P_w + \rho_w g z \]

Water flux

\[ q = -K h^\alpha (\nabla \phi)^\beta \]

Evolution equation for element(s)

\[ \frac{\partial h'}{\partial t} = \text{opening} - \text{closure} \]

Gwenn Flowers
Conservation of mass (continuity) in a 1-D water sheet or film

For a 1-D subglacial water sheet of thickness \( h = h(x, t) \) that varies in time \( t \) and in space \( x \), the volume per unit width of water in a sheet of length \( l \) is

\[
V = \int_0^l h(x, t) \, dx.
\]  

(6)

The rate of change of total water volume must be equal to the sum of sources and sinks, and can be written using the material derivative (see notes on continuum mechanics):

\[
\frac{dV}{dt} = \int_0^l \left( \frac{\partial h}{\partial t} + \frac{\partial (\bar{v} h)}{\partial x} \right) \, dx = \int_0^l (b + m) \, dx,
\]  

(7)

where \( \bar{v} \) is the fluid velocity, \( b \) represents the rate of water supply (from external sources such as the glacier surface or a groundwater system), \( m \) represents in-situ water production (e.g. from basal melt), and both \( b \) and \( m \) can be positive (sources) or negative (sinks). Defining the water flux in the standard way as \( q = \bar{v} h \), we can write the local form of the water balance as

\[
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = b + m.
\]  

(8)
Water sources, sinks, and fluxes

Figure from Martin Funk/Martin Lüthi/ETH-VAW Physics of Glaciers course notes
Governing equations for subglacial drainage

Continuity

\[ \frac{\partial h}{\partial t} + \nabla \cdot q = b \]

Source/sinks

\[ b = b_s + b_e + b_a + \frac{Q_G + Q_F}{\rho_i L} \]

Fluid potential

\[ \phi = P_w + \rho_w g z \]

Water flux

\[ q = -K h^\alpha (\nabla \phi)^\beta \]

Evolution equation for element(s)

\[ \frac{\partial h'}{\partial t} = \text{opening} - \text{closure} \]
Laminar and turbulent flow

Navier-Stokes equations
from Conservation of Momentum for motion of fluids
(Newton’s Second Law)

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \left( \mu \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right) - \frac{2}{3} \mu \left( \nabla \cdot \mathbf{u} \right) \mathbf{I} + \mathbf{F}
\]

- Inertial forces
- Pressure forces
- Viscous forces
- External forces

Reynolds number:
Used to predict laminar and turbulent flow regimes

\[
Re = \frac{\rho u L}{\mu}
\]

Low Re
- Inertial forces
- Viscous forces

Laminar flow

High Re

Turbulent flow
Laminar flow in sheet or film

Navier-Stokes equation
from Conservation of Momentum
(Newton’s Second Law)
Here assuming:
• Incompressible
• Laminar
• Parallel shear flow

Navier-Stokes equation
\[ \rho_w \frac{dv}{dt} = -\nabla P_w + \mu \nabla^2 v + \rho_w g, \]
Inertial forces
Viscous forces
Pressure forces
External forces

\[ \mu \frac{d^2 v_x}{dz^2} = \frac{d \phi}{dx}. \]
Viscous forces
Pressure forces

Poiseuille Flow
Assumptions:
• Steady-state
• Horizontal
• 1-d

Solve for velocity
• Integrate twice
• Apply appropriate boundary conditions

Solve for flux
• Integrate velocity with depth
• Apply appropriate boundary conditions

Solve for depth-averaged velocity

\[ \Phi = \text{hydraulic potential} \]
\[ \nu = \text{water velocity} \]
\[ \rho_w = \text{Density of water} \]
\[ \mu = \text{Dynamic viscosity of water} \]

\[ q_x = \int_0^h v_x(z) \, dz = \frac{d \phi}{dx} \frac{h^2}{2 \mu} \int_0^h \left( \frac{z^2}{h^2} - \frac{z}{h} \right) \, dz = -\frac{d \phi}{dx} \frac{h^3}{12 \mu}. \]
Laminar flow: \( q \propto \frac{d \phi}{dx} \)
Laminar flow in pipe or channel

Navier-Stokes equation
Radial coordinates

Solve for velocity
• Integrate twice
• Apply appropriate boundary conditions

\[ \frac{\mu}{r} \frac{d}{dr} \left( r \frac{dv}{dr} \right) = -\frac{d\phi}{ds}, \]

\[ v_s(r) = -\frac{d\phi}{ds} \frac{(R^2 - r^2)}{4\mu} \]
Porous media flow: Darcy’s Law

Porous media flow
- Creeping, laminar flow
- Tortuous paths
- Quantity of interest is not fluid velocity but effective velocity

Relevant to:
- Uneven water films
- Bulk behavior of linked cavity system (“microporous sheet”)
- Till canals
- Nye channels
- Drainage through till

\[ q = -Kh \frac{d\phi}{dx} \]

Flux
Hydraulic conductivity
Hydropotential gradient
Layer thickness

Ian Hewitt
Turbulent flow in pipe or channel

- Can be derived from Navier-Stokes equations
- More intuitive: balance of driving pressure force and resistive viscous force

\[
\frac{\partial \phi}{\partial s} \cdot \pi R^2 = 2\pi R \cdot \tau_{wall}
\]

Wall stress:
Empirical from engineering hydraulics
- Darcy-Weisbach
- Gauckler-Manning-Strickler

\[
\tau_{wall} = \frac{1}{8} f_R \rho_w \bar{v}^2
\]

Friction factor
\[
f_R = \frac{8 g \nu^2}{R_H^{1/3}}
\]

Combine to solve for velocity:

\[
\bar{v} = -\left(\frac{4 R}{f_r \rho_w}\right)^{1/2} \frac{\partial \phi}{\partial s} \left| \frac{\partial \phi}{\partial s} \right|^{-1/2}
\]

Turbulent flow: \( q \propto \frac{d\phi}{dx}^{1/2} \)
Governing equations for subglacial drainage

**Continuity**
\[
\frac{\partial h}{\partial t} + \nabla \cdot q = \dot{b}
\]

**Source/sinks**
\[
\dot{b} = \dot{b}_s + \dot{b}_e + \dot{b}_a + \frac{Q_G + Q_F}{\rho_i L}
\]

**Fluid potential**
\[
\phi = P_w + \rho_w g z
\]

**Water flux**
\[
q = -K h^\alpha (\nabla \phi)^\beta
\]

**Evolution equation for element(s)**
\[
\frac{\partial h'}{\partial t} = \text{opening} - \text{closure}
\]
Evolution of conduit (channel or cavity) volume

Conduit Evolution:
\[
\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |u_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)
\]

Energy balance for melt:
\[
mL = G + u_b \cdot \tau_b - q \cdot \nabla \phi
\]

- Melt opening
- Sliding over bumps
- Creep closure of ice
- Frictional heating
- Geothermal heat flux
- Viscous dissipation (laminar or turbulent)

Model output: sheet thickness, water pressure

Modified from Anderson, et al. 2004
Melt opening of a channel

Viscous dissipation
- power per unit length of conduit
- release of potential energy (gravitational + pressure)

Pressure-melt dependence
- when pressure varies
- some power required to keep water at melt temperature

Net power

As a melt rate
- Instantaneous heat transfer
- Temperate ice
- More complex treatment using full energy balance
Creep closure for a channel

Glen’s flow law
• Radial coordinates

\[ \frac{1}{R} \frac{dR}{dt} = A \left( \frac{p_i - p_w}{n} \right)^n \]

Rewrite as area change
\[ \frac{\partial S}{\partial t} \bigg|_{\text{creep closure}} = 2 S A \left( \frac{N}{n} \right)^n \]

Complexities
• Conduit shape – channels can be low and broad, non-channel shapes
  • Addition of shape factor
• Effective pressure typically substituted for normal stress
Classic conduit evolution mechanisms

**Opening**
- **Sliding**
  - YES
  - (but other mechanisms also possible)

**Melting**
- Passive sources
  - Maybe
- Viscous dissipation
  - No
  - (can lead to channelization)

**Closing**
- **Creep**
  - Yes
  - (but other mechanisms also possible)

**Channelized Drainage**
- No/not important
  - (but maybe can destroy)

**Distributed Drainage**
- No/not important
- YES
  - (can also cause closing)

Image: Ian Hewitt
Image: Tim Creyts
Classic conduit evolution mechanisms

Conduit Evolution:

\[
\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |u_b| \frac{h_r - h}{l_r} \right) - \left( \frac{h N}{\eta_i} \right)
\]

Energy balance for melt:

\[
mL = G + u_b \cdot \tau_b - q \cdot \nabla \phi
\]

Image: Ian Hewitt
Distributed Drainage

Image: Tim Creyts
Channelized Drainage

Frictional heating

Viscous dissipation
(laminar or turbulent)

Geothermal heat flux

\[
mL = G + u_b \cdot \tau_b - q \cdot \nabla \phi
\]

Frictional heating

Viscous dissipation
(laminar or turbulent)

Geothermal heat flux

Melt opening
Sliding over bumps
Creep closure of ice

Melt opening
Sliding over bumps
Creep closure of ice
Governing equations for subglacial drainage

Continuity

\[ \frac{\partial h}{\partial t} + \nabla \cdot q = b \]

Source/sinks

\[ b = b_s + b_e + b_a + \frac{Q_G + Q_F}{\rho_i L} \]

Fluid potential

\[ \phi = P_w + \rho_w g z \]

Water flux

\[ Q = \bar{v} S \quad \bar{v} = -\left( \frac{4 S^{1/2}}{f_r \rho_w \pi^{1/2}} \right)^{1/2} \frac{\partial \phi}{\partial s} \left| \frac{\partial \phi}{\partial s} \right|^{-1/2} \]

Evolution equation for element(s)

\[ \frac{\partial S}{\partial t} = \frac{Q}{\rho_i L} \left[ -\rho_w g \frac{\partial z_b}{\partial s} - (1 - \gamma) \frac{\partial p_w}{\partial s} \right] - 2 S A \left( \frac{p_i - p_w}{n} \right)^n \]
Putting it together: implications

Tendency for large channels to capture water from small ones in steady state.

Gwenn Flowers
Implications: channel vs. distributed drainage efficiency

Fig. 3. Typical values of effective pressure as a function of discharge for a steady state drainage system consisting of either a channel ($N_e$, dashed) or a distributed system ($N_{e_c}$, solid) across a basin of width 10 km with potential gradient $\Psi = 9$ Pa m$^{-1}$. 
Observations

Theory

Modeling
Literature: models of subglacial drainage
Overview: models of subglacial drainage

1. Elements (1960s - 1980s)
2. Early models combination of 0-D and 1-D (1970s - 1990s)
3. "Next generation" glaciological (1990s ±)

Increasing model sophistication

Increasing spatial dimensionality
Elements of the subglacial drainage system

Fast | Efficient | Channelized

- R-channels
- Broad/low channels
- Nye channels

Legend:
- Ice
- Rock
- Sediment
- Water

Canals

- Sheets & films
- Cavities
- Porous flow

\[ Q \uparrow \Rightarrow P \downarrow \]

\[ Q \uparrow \Rightarrow P \uparrow \]

Diagrams courtesy of Tim Creyts
Elements of the subglacial drainage system

- Drainage system morphology defined by presence and interaction of different elements
- Observed and inferred subglacial drainage dynamics implies spatial and temporal variations in system morphology

Overview: Models of subglacial drainage

Recipe for a model of subglacial drainage

Flux

$$q = -K h^\alpha (\nabla \phi)^\beta$$

Fluid potential

$$\phi = P_w + \rho_w g z$$

Continuity

$$\frac{\partial h}{\partial t} + \nabla \cdot q = \dot{b}$$

Sources/sinks

$$\dot{b} = \dot{b}_s + \dot{b}_e + \dot{b}_a + \frac{Q_G + Q_F}{\rho_i L}$$

Evolution equation for element(s)

$$\frac{\partial h'}{\partial t} = \text{opening} - \text{closure}$$

Gwenn Flowers
Early models from groundwater hydrology (1970s – 1990s)

Flux

\[ q = -K \nabla \phi \]

Darcy’s Law

Fluid potential

\[ \phi = P_w + \rho_w g z \]

Continuity

\[ \nabla \cdot q = b \]

Steady state

Sources/sinks

\[ b = \frac{Q_G + Q_F}{\rho_i L} \]

Basal melt

Gwenn Flowers
Motivated by an interest in basal till deformation, fast flow, fate of basal melt, impact of glaciation on groundwater

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model dimension</th>
<th>Model domain</th>
<th>Steady-state / transient</th>
<th>Drainage at ice-bed interface?</th>
<th>Water sources</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campbell &amp; Rasmussen (1973)</td>
<td>1D (x)</td>
<td>Glacier body</td>
<td>transient</td>
<td>No</td>
<td>Calculated surface melt</td>
<td>South Cascade Glacier (USA)</td>
</tr>
<tr>
<td>Shoemaker (1986)</td>
<td>1D (y)</td>
<td>Deformable aquifer over aquitard</td>
<td>steady state</td>
<td>Prescribed R channels</td>
<td>Prescribed basal melt</td>
<td>idealized transect</td>
</tr>
<tr>
<td>Shoemaker &amp; Leung (1987)</td>
<td>2D (y-z in aquifer, z in aquitard)</td>
<td>Till aquitard over aquifer</td>
<td>steady-state</td>
<td>Prescribed R channels</td>
<td>Prescribed basal melt</td>
<td>idealized transect</td>
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<td>Lingle &amp; Brown (1987)</td>
<td>1D (x)</td>
<td>Subglacial aquifer</td>
<td>steady state</td>
<td>No</td>
<td>Calculated basal melt</td>
<td>Ice Stream B, Antarctica</td>
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<td>Boulton et al. (1993)</td>
<td>2D (x-z)</td>
<td>Aquifers separated by aquitard</td>
<td>steady state</td>
<td>No</td>
<td>Prescribed basal melt, upstream flux</td>
<td>Saalian ice sheet transect, Netherlands</td>
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<td>Modelled basal melt</td>
<td>European ice sheet flowline, Sweden to Germany</td>
</tr>
<tr>
<td>Piotrowski (1997)</td>
<td>3D</td>
<td>Multi-layer subsurface</td>
<td>steady state</td>
<td>Tunnel valley outburst floods</td>
<td>Prescribed basal melt, upstream flux</td>
<td>Scandinavian ice sheet sector, NW Germany</td>
</tr>
</tbody>
</table>
First spatially resolved model of glacier drainage system?

Campbell and Rasmussen (1973): The production, flow and distribution of melt water in a glacier treated as a porous medium, *Hydrology of Glaciers*


Fig. 9. The vertically integrated transport $Q$ in $10^6$ m$^3$/h (solid lines) and the change in thickness of the saturated layer in cm/2 h (dashed lines) for the actual bed with $K_t=0.0015$ and $K_s=0.00001$.
Motivated by an interest in basal till deformation, fast flow, fate of basal melt, impact of glaciation on groundwater.

Hydrogeologic units beneath ice sheets rarely capable of evacuating even basal melt from ice sheets.

Some interfacial drainage system required, though rarely formalized.

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<td>Prescribed basal melt</td>
<td>idealized transect</td>
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<tr>
<td>Shoemaker &amp; Leung (1987)</td>
<td>2D (y–z) in aquifer, z in aquitard</td>
<td>Till aquitard over aquifer</td>
<td>steady-state</td>
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<td>Calculated basal melt</td>
<td>Ice Stream B, Antarctica</td>
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<td>Aquifers separated by aquitard</td>
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<td>Tunnel valley outburst floods</td>
<td>Prescribed basal melt, upstream flux</td>
<td>Scandinavian ice sheet sector, NW Germany</td>
</tr>
</tbody>
</table>
Increasing model sophistication

1. Elements (1960s - 1980s)

2. Early growth models (1990s ±)

3. “Next generation” glaciological models (1990s ±)

4. Multi-element models (1990s ±)

Increasing spatial dimensionality

0-D

1-D

2-D

3-D
“Next-generation” glaciological models (1990s ±)

Focus on drainage at the ice-bed interface
Models employ more of the theoretical drainage elements

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model dimension</th>
<th>Steady-state / transient</th>
<th>Drainage morphology</th>
<th>Water sources</th>
<th>Coupling to dynamics</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budd &amp; Jenssen (1987)</td>
<td>2D ($x$-$y$)</td>
<td>steady state</td>
<td>Laminar film</td>
<td>Modelled basal melt</td>
<td>Yes</td>
<td>West Antarctic Ross Ice Shelf Basin</td>
</tr>
<tr>
<td></td>
<td>1D ($x$), 1992; 2D ($x$-$y$), 2002</td>
<td>steady state</td>
<td>Cavities or R-channels</td>
<td>Modelled basal melt, surface melt</td>
<td>Yes</td>
<td>Weichselian Scandinavian ice sheet</td>
</tr>
<tr>
<td>Arnold &amp; Sharp (1992, 2002)</td>
<td>1D ($y$)</td>
<td>transient</td>
<td>Laminar film</td>
<td>–</td>
<td>Yes</td>
<td>Idealized mountain glacier</td>
</tr>
<tr>
<td>Alley (1996)</td>
<td></td>
<td>transient</td>
<td>R-channel network</td>
<td>Modelled moulin input</td>
<td>No</td>
<td>Haut Glacier d’Arolla, Switzerland</td>
</tr>
<tr>
<td>Arnold et al. (1998)</td>
<td>2.5D</td>
<td>transient</td>
<td>Macroporous sheet</td>
<td>Modelled basal melt, englacial / groundwater exchange</td>
<td>No</td>
<td>Trapridge Glacier, Canada</td>
</tr>
<tr>
<td>Flowers &amp; Clarke (2002a,b)</td>
<td>2.5D</td>
<td>transient</td>
<td>Laminar / turbulent film</td>
<td>Modelled basal melt</td>
<td>Yes</td>
<td>Northern hemisphere ice sheets</td>
</tr>
<tr>
<td>Johnson &amp; Fastook (2002)</td>
<td>2D ($x$-$y$)</td>
<td>transient</td>
<td>Macroporous sheet</td>
<td>Modelled basal melt</td>
<td>Yes</td>
<td>Northern hemisphere ice sheets</td>
</tr>
</tbody>
</table>

Gwenn Flowers
Most ice-sheet models adopt a Weertman-type laminar water film (e.g. Budd and Jenssen, 1987; Johnson and Fastook, 2002)

Flux

\[ q = -\frac{\nabla \phi \, h^3}{12 \mu} \]

Fluid potential

\[ \phi = \rho_i \, g \, H + \rho_w \, g \, z \]

Continuity

\[ \frac{\partial h}{\partial t} + \nabla \cdot q = \dot{b} \]

Sources/sinks

\[ \dot{b} = \frac{Q_G + Q_F}{\rho_i \, L} \]

Sliding often related to film thickness (N=0 often assumed)
Basal water film thickness (mm) calculated beneath the Ross Sea ice streams, West Antarctica

Budd and Jenssen (1987): Numerical modelling of the large-scale basal water flux under the West Antarctic Ice Sheet, *Dynamics of the West Antarctic ice sheet*

LeBrocq et al. (2009): A subglacial water-flow model for West Antarctica: *Journal of Glaciology*
Channel network model for Haut Glacier d’Arolla, Switzerland

Arnold et al., 1998: Initial results from a semi-distributed, physically based model of glacier hydrology, *Hydrological Processes*
Increasing model sophistication

- **0-D**:
  - 1. Elements (1960s - 1980s)

- **1-D**:  
  - 2. Early models from groundwater hydrology (1970s - 1990s)
  - 3. "Next generation" glaciological models (1990s ±

- **2-D**:
Integrating multiple drainage elements (2000s – present)

Appropriate physics for fast vs. slow systems

\[ q_{\text{fast/slow}} = -K \ h^\alpha \ (\nabla \phi)^\beta \]

**Flux**

**Fluid potential**

\[ \phi = P_w + \rho_w \ g \ z \]

**Continuity**

\[ \frac{\partial h}{\partial t} + \nabla \cdot q = \dot{b} \]

**Sources/sinks**

\[ \dot{b} = \dot{b}_s + \dot{b}_e + \dot{b}_a + \frac{Q_G + Q_F}{\rho_i L} \]

**Evolution equation for element(s)**

\[ \frac{\partial h'}{\partial t}_{\text{fast/slow}} = \text{opening} - \text{closure} \]

Gwenn Flowers
<table>
<thead>
<tr>
<th>Reference</th>
<th>Model dimension</th>
<th>Slow-system morphology</th>
<th>Fast-system morphology</th>
<th>Sliding feedback</th>
<th>Coupling to dynamics</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarke (1996)</td>
<td>lumped</td>
<td>various</td>
<td>various</td>
<td>No</td>
<td>No</td>
<td>Generalized glacier system</td>
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<tr>
<td>Arnold &amp; Sharp (1992)</td>
<td>1D</td>
<td>Cavities</td>
<td>R-channels</td>
<td>No</td>
<td>Yes</td>
<td>Scandinavian ice-sheet</td>
</tr>
<tr>
<td>Flowers et al. (2004)</td>
<td>1D</td>
<td>Laminar / turbulent sheet</td>
<td>R-channels</td>
<td>No</td>
<td>No</td>
<td>Skeidararjökull, Iceland</td>
</tr>
<tr>
<td>Kessler &amp; Anderson (2004)</td>
<td>1D</td>
<td>Cavitites + englacial storage</td>
<td>R-channel</td>
<td>Yes</td>
<td>Yes</td>
<td>Idealized glacier</td>
</tr>
<tr>
<td>Flowers (2008)</td>
<td>1D</td>
<td>Macroporous sheet</td>
<td>R-channels</td>
<td>No</td>
<td>No</td>
<td>Idealized glacier</td>
</tr>
<tr>
<td>Hewitt &amp; Fowler (2008)</td>
<td>1D</td>
<td>Cavities</td>
<td>R-channels</td>
<td>Yes</td>
<td>Yes</td>
<td>Idealized glacier</td>
</tr>
<tr>
<td>Colgan et al. (2011, 2012)</td>
<td>1D</td>
<td>Englacial storage</td>
<td>R-channel segments</td>
<td>No</td>
<td>Yes</td>
<td>Sermeq Avannarleq, Greenland</td>
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<tr>
<td>Pimentel &amp; Flowers (2011)</td>
<td>1D</td>
<td>Macroporous sheet</td>
<td>R-channels</td>
<td>No</td>
<td>Yes</td>
<td>Idealized outlet glaciers</td>
</tr>
<tr>
<td>Kingslake &amp; Ng (2013)</td>
<td>1D</td>
<td>Cavities</td>
<td>R-channels</td>
<td>Yes</td>
<td>Yes</td>
<td>Idealized mountain glacier</td>
</tr>
<tr>
<td>Schoof et al. (2013)</td>
<td>1D</td>
<td>Cavities</td>
<td>R-channels</td>
<td>No</td>
<td>No</td>
<td>Idealized mountain glacier</td>
</tr>
<tr>
<td>de Fleurian et al. (2014)</td>
<td>1D</td>
<td>Porous medium</td>
<td>Porous medium</td>
<td>No</td>
<td>Yes</td>
<td>Haut Glacier d’Arolla</td>
</tr>
<tr>
<td>Arnold &amp; Sharp (2002)</td>
<td>2D</td>
<td>Cavities (1 per cell)</td>
<td>R-channels</td>
<td>No</td>
<td>Yes</td>
<td>Scandinavian ice-sheet</td>
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<tr>
<td>Schoof (2010)</td>
<td>2D</td>
<td>Cavities</td>
<td>R-channels (2D)</td>
<td>No</td>
<td>No</td>
<td>Idealized ice-sheet margin</td>
</tr>
<tr>
<td>Hewitt (2011)</td>
<td>2D</td>
<td>Effective porous medium</td>
<td>R-channels (1D)</td>
<td>No</td>
<td>No</td>
<td>Idealized ice-sheet margin</td>
</tr>
<tr>
<td>Hewitt (2013)</td>
<td>2D</td>
<td>Cavities</td>
<td>R-channels (2D)</td>
<td>Yes</td>
<td>Yes</td>
<td>Idealized ice-sheet margin</td>
</tr>
<tr>
<td>Werder et al. (2013)</td>
<td>2D</td>
<td>Cavities</td>
<td>R-channels (2D)</td>
<td>No</td>
<td>No</td>
<td>Idealized ice-sheet margin, Gornergletscher</td>
</tr>
<tr>
<td>Hoffman &amp; Price (2014)</td>
<td>2D</td>
<td>Cavities</td>
<td>R-channel (1D)</td>
<td>Yes</td>
<td>Yes</td>
<td>Idealized mountain glacier</td>
</tr>
</tbody>
</table>
Conceptual model: Röthlisberger channels interacting with a distributed drainage system (e.g. cavities or sheet)

<table>
<thead>
<tr>
<th>Distributed system</th>
<th>R-channels</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
</tbody>
</table>

### Flux / discharge

- \( q \propto h \nabla \phi \)
- \( q \propto h^{3/2} \sqrt{\nabla \phi} \)
- \( Q \propto S^{5/2} \sqrt{\frac{\nabla \phi}{f_R}} \)

### Evolution equation for elements

\[
\frac{\partial h}{\partial t} = \frac{\partial h}{\partial t}(u_b, hAN^n)
\]

\[
\frac{\partial S}{\partial t} = \frac{\partial S}{\partial t}(\dot{M}, hAN^n)
\]

### Coupling with ice dynamics

- \( \rightarrow u_b = u_b(N, h) \)
- \( \leftarrow \frac{\partial h}{\partial t} = \frac{\partial h}{\partial t}(u_b) \)

Gwenn Flowers
1D conceptual models:

Sheet-channel system

Cavity-channel system


Kingslake and Ng (2013): Modelling the coupling of flood discharge with glacier flow during jokulhlaups, *Annals of Glaciology*
Coupled models allow more realistic simulation of seasonal transitions

- Sliding feedback usually absent in sheet-based distributed system

Cavity-based system captures sliding feedback (two-way coupling)

\[ u_b = u_b(N) \]
\[ \frac{\partial h}{\partial t} = \frac{\partial h}{\partial t}(u_b) \]

2-D network of cavity elements with spontaneous channel evolution

2-D cavity system with spontaneous channel evolution along mesh edges.


Werder et al. (2013): Modeling channelized and distributed subglacial drainage in two dimensions, *Journal of Geophysical Research*
Subglacial drainage model zoo

Subglacial Hydrology Model Intercomparison Project (SHMIP) de Fleurian et al. *in review*, J. Glac.
Features of current models

- Correct (?) physics applied to fast and slow drainage systems
- Numerical formulation (2D distributed system, network of 1D channel elements)
- Dynamic evolution of drainage system
- Two-way coupling to sliding/friction laws

Modelling challenges and limitations

- Resolution of individual channel elements
- Assumption of fully “connected” drainage system
- Temperate bed assumption
- Dearth of calibration/validation data, resolution of input data
Too complicated?

Do we know enough to pursue increased model sophistication?

Diffusion of basal melt in PISM (Bueler and Brown, 2009):

$$\frac{\partial h}{\partial t} = b + K_{\text{melt}} \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right)$$

$$K_{\text{melt}} = \frac{\bar{L}}{2\bar{t}^2}$$

$$P_w = 0.95 (\rho_i g H) \left( \frac{h}{2m} \right)$$

What computational burden is acceptable?

“Double continuum” drainage model (de Fleurian et al., 2014)

What models are applicable to the continental scale?

Flow routing algorithms (e.g. Carter et al., 2011; Goeller et al., 2013)
Field observations more complex and richer than current models can explain

• Out of phase borehole pressure variations
• Water pressure above floatation pressure
• Large pressure gradients between neighboring boreholes
• Switching between “connected” and “disconnected” behavior
• Very high winter water pressure despite negligible water input
• ...

Rada and Schoof (2018) Subglacial drainage characterization from eight years of continuous borehole data on a small glacier in the Yukon Territory, Canada. The Cryosphere Discussions.
Ice dynamics responds to integrated basal traction over *entire* bed.


Rada and Schoof (2018) Subglacial drainage characterization from eight years of continuous borehole data on a small glacier in the Yukon Territory, Canada. The Cryosphere Discussions.
Recommendations for ongoing and future work

• Efficient representation of drainage networks in large-scale models
• Continuum approached for efficient drainage that converge with grid resolution
• Alternative approaches to continuum models or explicit treatment of “connected” and “unconnected” bed areas
• Unified treatment of hard and soft beds?
• More attention to polythermal conditions (c.f. e.g. Bueler and Brown, 2009; Creyts and Clarke, 2010)
• Surface, englacial, groundwater drainage
• Develop methods to “measure” basal water pressure at scales commensurate with ice dynamics