EQUILIBRIUM LINE ALTITUDE (ELA)

- ELA (variable) is part of the annual balance
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- Degree-day model
CORRELATION OF ELA AND ANNUAL BALANCE FOR HINTEREISFERNER

- Hintereisferner
- Year-to-year fluctuations are correlated
- Approximately linear relation
- Balance-budget ELA when balance = 0
SIMPLE MODEL
(Braithwaite & Raper, 2009)

• \( b_t = \beta \cdot (ELA_t - ELA_0) \)  \hspace{1cm} (1)
Where \( b_t \) is mean specific balance for year \( t \), \( ELA_t \) is ELA for year \( t \), \( ELA_0 \) is balanced budget ELA, and \( \beta \) is empirical factor for each glacier.

• \( b_{it} = k \cdot (h_i - ELA_t) \)  \hspace{1cm} (2)
Where \( b_{it} \) is specific balance at altitude \( h_i \) in year \( t \), and \( k \) is balance gradient near the ELA.

• \( b_t = \frac{1}{A} \sum A_i \cdot b_{it} \)  \hspace{1cm} (3)
Where \( A_i \) is area around altitude \( h_i \) and \( A \) is total area.

• \( b_t = -k \cdot (ELA_t - h_{mn}) \)  \hspace{1cm} (4)
Where \( h_{mn} \) is the area-weighted mean altitude of the glacier.

• \( k = -\beta \)  \hspace{1cm} (5)

Balance gradient for Hintereisferner is c. 0.5 m w.e./100 m altitude. Equality of balanced-budget ELA and mean altitude first stated by Kurowski (1891) if we allow for changes in terminology.
• Hintereisferner has long MB record
• Calculate balanced-budget ELA for overlapping 5-year chunks of data
• Balance budget ELA varies but varies much less than ELA
EXTENDING THE KUROWSKI MODEL

- If area altitude distribution is symmetric around mean altitude $h_{mn}$
- Median altitude $h_{med} = h_{mn}$
- Accumulation area ratio (AAR) = 0.5 for $h_{med}$
- AAR = 0.5 often assumed for balanced-budget ELA
- Balance gradient is not constant, suggesting that AAR > 0.5
- AAR = 0.67 suggested for alpine glaciers
CORRELATION OF ELA AND ANNUAL BALANCE FOR OTHER GLACIERS
(Braithwaite & Raper, 2009)

Data available August 2008. Needs updating!

Correlations for series > 5 years.
Note poor correlation for some glaciers. Presently unexplained!

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High correlation (on left) is forced by geographical variations
Lower correlations with normalized data
### ESTIMATION OF BALANCED-BUDGET ELA

(Braithwaite & Raper, 2009)

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean error</th>
<th>STD error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median altitude (94 glaciers)</td>
<td>-38 m</td>
<td>± 82 m</td>
</tr>
<tr>
<td>Mid-point altitude (94 glaciers)</td>
<td>-27 m</td>
<td>± 125 m</td>
</tr>
</tbody>
</table>

- Estimate balanced-budget ELA from median altitude $h_{med}$ with error of c. ± 80 m
- If you don’t know the median altitude, you can use the mid-point altitude $(h_{max} - h_{min})/2$ with a greater error of c. ± 130 m
NUMBERS
(Braithwaite & Raper, 2009)

Graphs 7-9 based on August 2008 dataset and can be updated anytime. In August 2008:
• MB data available for 351 glaciers
• 127 glaciers had > 4 years of MB-ELA record to calculate ELA₀
• 116 out of 127 glaciers had MB-ELA correlation < -0.71 to calculate reliable ELA₀
• 94 out of 116 glaciers had data h_med to compare with ELA₀

An update to 2010 will probably increase these numbers a little
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TEMPERATURE LAPSE RATE

- ELA (a.k.a. snow line) and temperature lapse rate (TLR) have been linked for a long time
- TLR varies with season and latitude
- Models often use constant lapse rate of c. 0.6 K/100 m
• Meteoswiss have published climatic norms for 1961-1990 for many stations
• We can plot summer (June-August) mean temperature versus altitude
• Strong linear correlation over large height range
SEASONAL VARIATION IN TLR
(Braithwaite, unpublished)

TLR for June-August somewhat less than 0.6 deg/100 m

Use regression lines for each month to calculate monthly TLR
TEMPERATURE ANOMALIES
(Braithwaite, unpublished)

• Low and high stations in and around the Alps have remarkably similar patterns
• Suggests that TLR may not be much affected by climate change
• This point is worth watching!

The Alps

<table>
<thead>
<tr>
<th>Year (summer)</th>
<th>Summer mean temperature anomaly (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>-2.5</td>
</tr>
<tr>
<td>1970</td>
<td>-1.8</td>
</tr>
<tr>
<td>1980</td>
<td>0.0</td>
</tr>
<tr>
<td>1990</td>
<td>2.3</td>
</tr>
<tr>
<td>2000</td>
<td>4.0</td>
</tr>
<tr>
<td>2010</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Legend:
- 3 high stations
- 10 low stations
GLACIER COOLING EFFECT - 1
(Braithwaite, unpublished unsolved problem)

Very similar (but not same?) cooling effect at these two Greenland glaciers
Under climate change, glacier temperature will not rise as quickly as regional
temperature

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GLACIER COOLING EFFECT - 2
(Braithwaite, unpublished unsolved problem)

- Could there be a change of slope?
- For low temperatures glacier surface will be same temperature as snow-free ground
- More work needed!
- For higher temperatures glacier surface will be fixed at 0 C
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Ohmura et al. (1992) gives winter balance and “winter balance + summer precipitation”. Braithwaite (2008) interprets these as low (Version 1) and high (Version 2) estimates of annual accumulation (unmeasurable).
The logic, going back to Ahlmann (1922), is that (1) snowmelt is a nonlinear function of summer temperature, (2) snowmelt = snow accumulation at ELA, (3) snow accumulation is nonlinear function of summer temperature. Exponent and power law relations have been proposed.
MEAN TEMPERATURE AT ELA

- Ohmura et al (1992) gives summer (June-August) mean temperature at ELA
- If monthly temperatures at ELA follow sine wave (of known range) we can calculate monthly temperatures
- Get temperature amplitude from CRU gridded climatology
ICE ABLATION AND TEMPERATURE
(Braithwaite, 1981, 1996 & 2011)

Well-constrained relationship between ice ablation and air temperature from Arctic & Greenland
DEGREE-DAY MODEL

Positive degree-day factors for ablation on the Greenland ice sheet studied by energy-balance modelling

Roger J. Brathwaite
Grenland Geologiske Undersøgelse, DK-3000 København K, Danmark

Degree-day model well justified by energy-balance

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The degree-day sum for a period of $N$ days is given by:

$$ Y = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{mn} T_{nm} $$  \hspace{1cm} (1) $$

where $\alpha_{mn}$ has either a value of unity or zero according to:

$$ \alpha_{mn} = 1.0 \text{ if } T_{nm} > 0 \degree C \quad = 0.0 \text{ if } T_{nm} < 0 \degree C \quad (2) $$

$T_{nm}$ is the temperature at the $m$th observation on the $n$th day and $M$ is the number of observations in the day.

If the temperature is assumed to constitute a stationary random series, the time-summation in Equation (1) can be replaced by an ensemble-summation as follows:

$$ Y = N \sum_{k=0}^{K} f(T_k) T_k $$  \hspace{1cm} (3) $$

where:

$$ T_k = T_0 + k \Delta T $$  \hspace{1cm} (4) $$

and $f(T_k)$ is the probability that the temperature lies in an interval of width $\Delta T$ centred on $T_k$ and $K$ is the value of $k$ such that $f(T_k)$ becomes zero. $T_0$ is zero for the computation of positive degree-days.

- Strictly, degree-day totals are calculated from daily temperature data
- Can calculate degree-day totals from monthly data using probability model
- Assume monthly temperatures are normally distributed with specified standard deviation
Use Braithwaite (1985) model to calculate monthly melt and monthly accumulation

For very small annual temperature range, e.g. in tropics, “summer” mean temperature cannot be much more than 0°C. This is because we need a “winter” with below 0°C temperature to allow snow formation.
CLIMATE AT ELA AGAIN
(Ohmura et al. 1992 and Braithwaite, 2008)

- We can now “explain” the accumulation-temperature relation
- Different curves for different annual temperature range

![Graph showing annual accumulation at ELA (m water a⁻¹) vs. summer mean temperature at ELA (°C)]

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So now you know why accumulation at ELA depends on temperature.