Thermodynamics of Glaciers

McCarthy Summer School, June 2010

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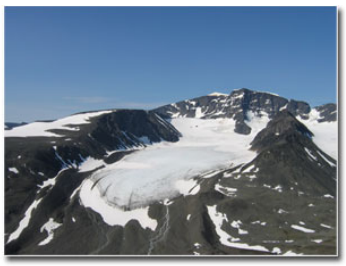
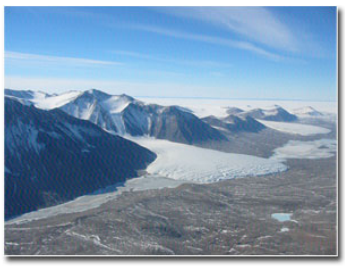
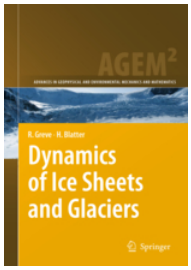
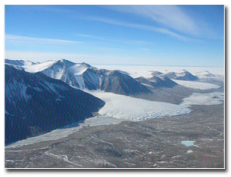



On Notation

- (hopefully) consistent with Continuum Mechanics (Truffer)
- with lots of input from *Luethi & Funk: Physics of Glaciers I* lecture at ETH
- notation following Greve & Blatter: Dynamics of Ice Sheets and Ice Sheets



Types of Glaciers

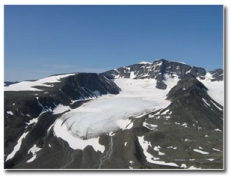


cold glacier

ice below pressure melting point, no liquid water

temperate glacier

ice at pressure melting point, contains liquid water in the ice matrix



polythermal glacier

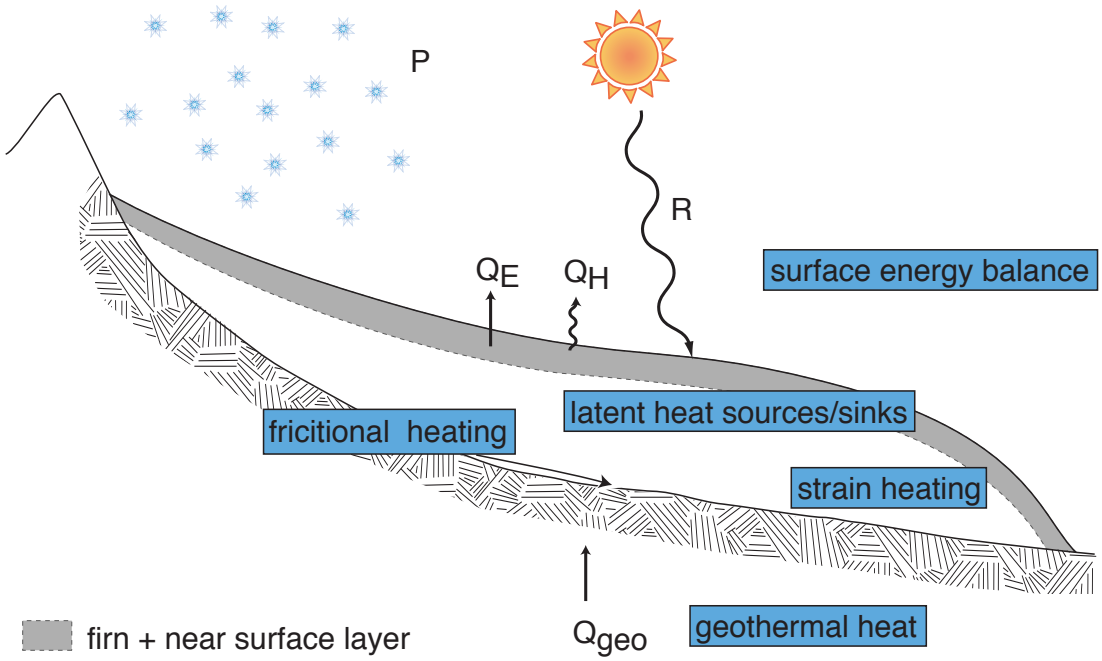
cold and temperate parts

Why we care

The knowledge of the distribution of temperature in glaciers and ice sheets is of high practical interest

- A temperature profile from a cold glacier contains information on past climate conditions.
- Ice deformation is strongly dependent on temperature (temperature dependence of the rate factor A in Glen's flow law);
- The routing of meltwater through a glacier is affected by ice temperature. Cold ice is essentially impermeable, except for discrete cracks and channels.
- If the temperature at the ice-bed contact is at the pressure melting temperature the glacier can slide over the base.
- Wave velocities of radio and seismic signals are temperature dependent. This affects the interpretation of ice depth soundings.

Energy balance: depicted



Energy balance: equation

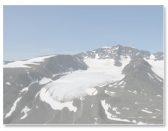
$$\rho\left(\frac{\partial u}{\partial t} + \mathbf{v}\cdot\nabla u\right) = -\nabla\cdot\mathbf{q} + Q$$

ρ	ice density
u	internal energy
$\mathbf{v}$	velocity
$\mathbf{q}$	heat flux
Q	dissipation power (strain heating)

Noteworthy

- strictly speaking, internal energy is not a conserved quantity
- only the sum of internal energy and kinetic energy is a conserved quantity

Temperature equation



Temperature equation

- ice is cold if a change in heat content leads to a change in temperature alone
- independent variable: temperature $T = c(T)^{-1}u$

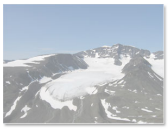
$$\rho c(T)\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = -\nabla\cdot\mathbf{q} + Q$$

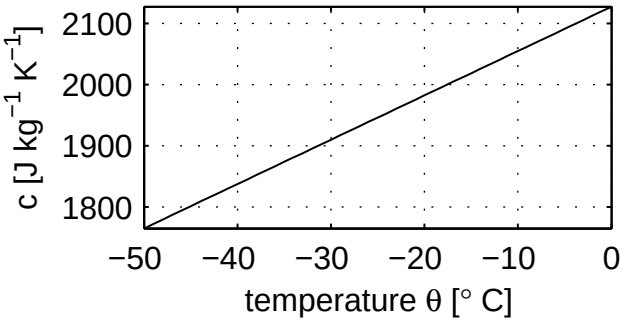
Fourier-type sensible heat flux

$$\mathbf{q} = \mathbf{q}_s = -k(T)\nabla T$$

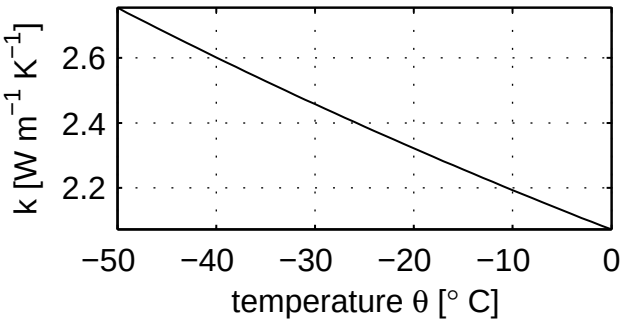
$c(T)$	heat capacity
$k(T)$	thermal conductivity

Thermal properties



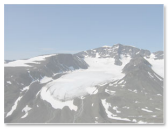


heat capacity is a monotonically-increasing function of temperature



thermal conductivity is a monotonically-decreasing function of temperature

Flow law

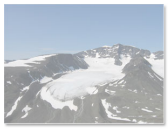


Viscosity η is a function of effective strain rate d_e and temperature T

$$\eta = \eta(T, d_e) = 1/2B(T)d_e^{(1-n)/n}$$

where $B = A(T)^{-1/n}$ depends exponentially on T

Ice temperatures close to the glacier surface



Assumptions

- only the top-most 15 m experience seasonal changes
- heat diffusion is dominant

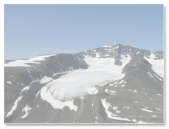
We then get

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial h^2}$$

where h is depth below the surface, and $\kappa = k/(\rho c)$ is the thermal diffusivity of ice

Ice temperatures close to the glacier surface

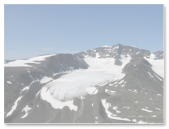
Boundary Conditions

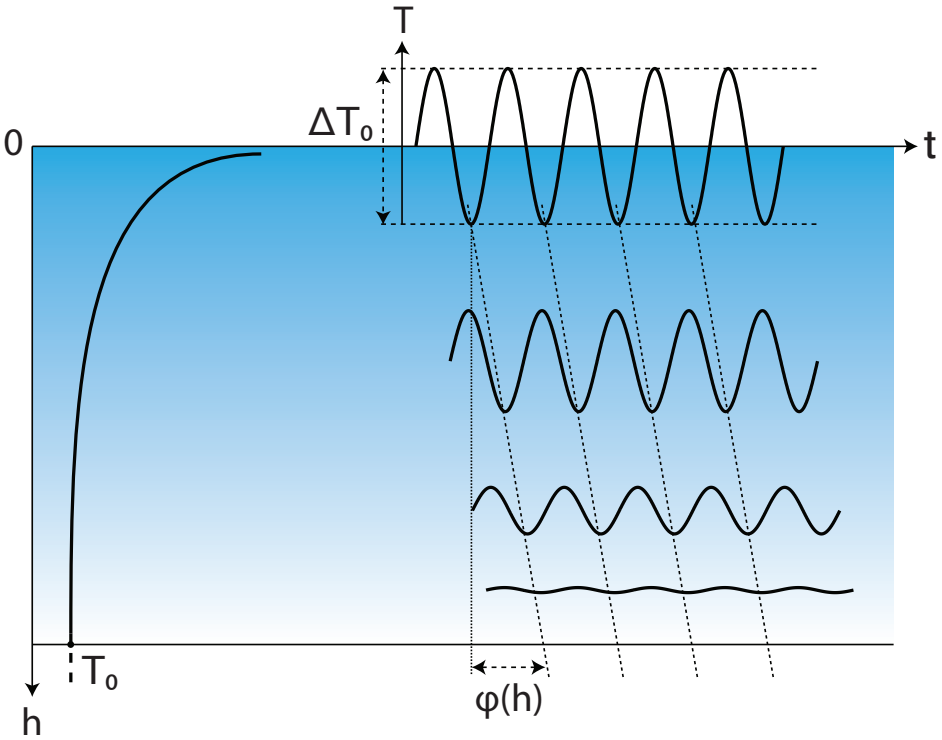


$$T(0,t) = T_0 + \Delta T_0 \cdot \sin(\omega t)\,,$$
$$T(\infty,t) = T_0\,.$$

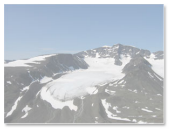
T_0	mean surface temperature
ΔT_0	amplitude
$2\pi/\omega$	frequency

Ice temperatures close to the glacier surface





Ice temperatures close to the glacier surface

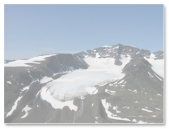


Analytical Solution

$$T(h,t) = T_0 + \underbrace{\Delta T_0 \exp\left(-h\sqrt{\frac{\omega}{2\kappa}}\right)}_{\Delta T(h)} \sin\left(\omega t - \underbrace{h\sqrt{\frac{\omega}{2\kappa}}}_{\varphi(h)}\right).$$

$\Delta T(h)$ amplitude variation with depth

Ice temperatures close to ice divides



Assumptions

- only vertical advection and diffusion

We then get

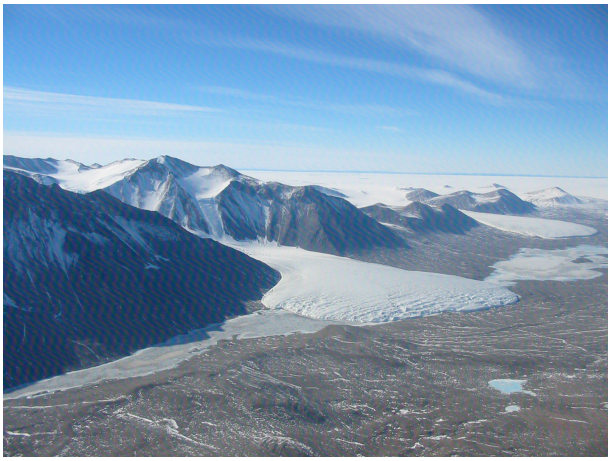
$$\kappa \frac{\partial^2 T}{\partial z^2} = w(z) \frac{\partial T}{\partial z}$$

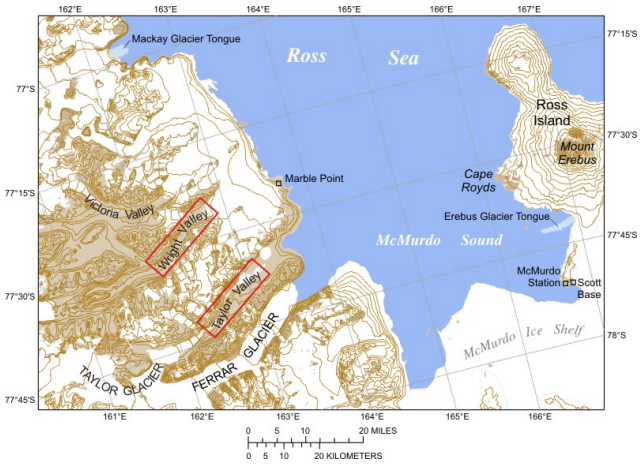
where w is the vertical velocity

Analytical solution

- can be obtained

Cold Glaciers

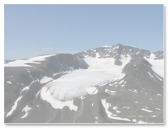




- Dry Valleys, Antarctica
- (very) high altitudes at lower latitudes

Water content equation

Water content equation

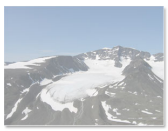


- Ice is temperate if a change in heat content leads to a change in water content alone
- independent variable: water content (aka moisture content, liquid water fraction) $\omega = L^{-1}u$

$$\rho L \left(\frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega \right) = -\nabla \cdot \mathbf{q} + Q$$

$\Rightarrow$ in temperate ice, water content plays the role of temperature

Flow law



Flow Law

Viscosity η is a function of effective strain rate d_e and water content ω

$$\eta = \eta(\omega, d_e) = 1/2B(\omega)d_e^{(1-n)/n}$$

where B depends linearly on ω

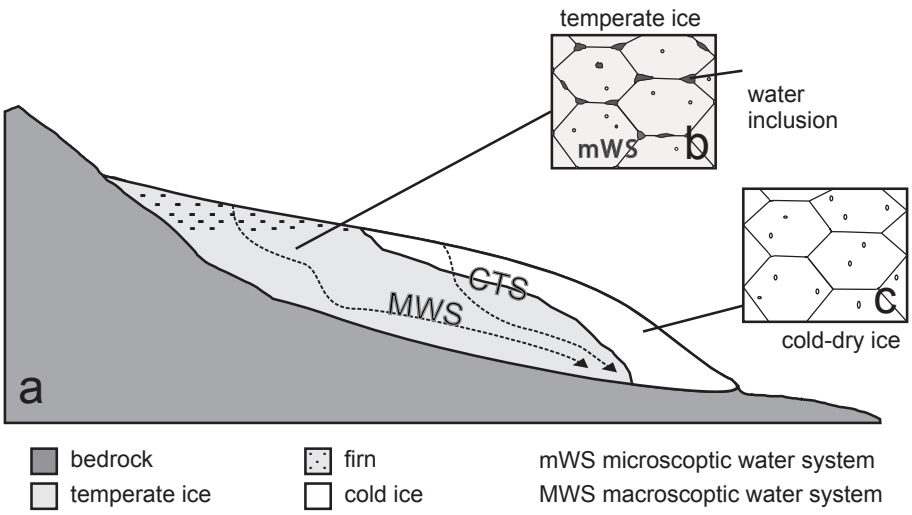
- but only very few studies (e.g. from Lliboutry and Duval)

Latent heat flux

$$\mathbf{q} = \mathbf{q}_l = \begin{cases} \text{Fick-type} \\ \text{Darcy-type} \end{cases}$$

$\Rightarrow$ leads to different mixture theories (*Class I*, *Class II*, *Class III*)

Sources for liquid water in temperate Ice



1. water trapped in the ice as water-filled pores
2. water entering the glacier through cracks and crevasses at the ice surface in the ablation area
3. changes in the pressure melting point due to changes in lithostatic pressure
4. melting due energy dissipation by internal friction (strain heating)

Temperature and water content of temperate ice

Temperature

$$T_m = T_{\mathrm{tp}} - \gamma \left(p - p_{\mathrm{tp}}\right), \tag{1}$$

- $T_{\mathrm{tp}} = 273.16\,\mathrm{K}$ triple point temperature of water
- $p_{\mathrm{tp}} = 611.73\,\mathrm{Pa}$ triple point pressure of water
- Temperature follows the pressure field

Water content

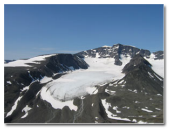
- generally between 0 and 3%
- water contents up to 9% found

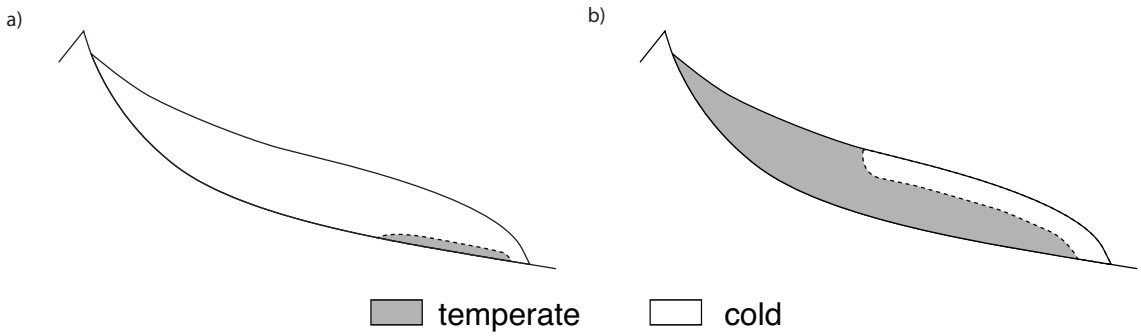
Temperate Glaciers

Temperate glaciers are widespread, e.g.:

- Alps, Andes, Alaska,
- Rocky Mountains, tropical glaciers, Himalaya

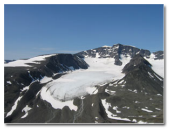
Polythermal glaciers

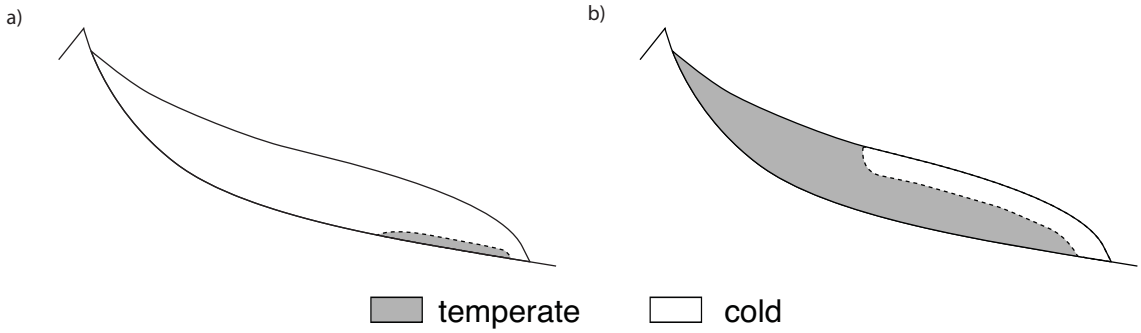




- contains both cold and temperate ice
- separated by the cold-temperate transition surface (CTS)
- CTS is an internal free surface of discontinuity where phase changes may occur
- polythermal glaciers, but not polythermal ice

Scandinavian-type thermal structure

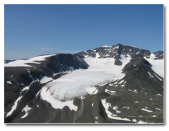


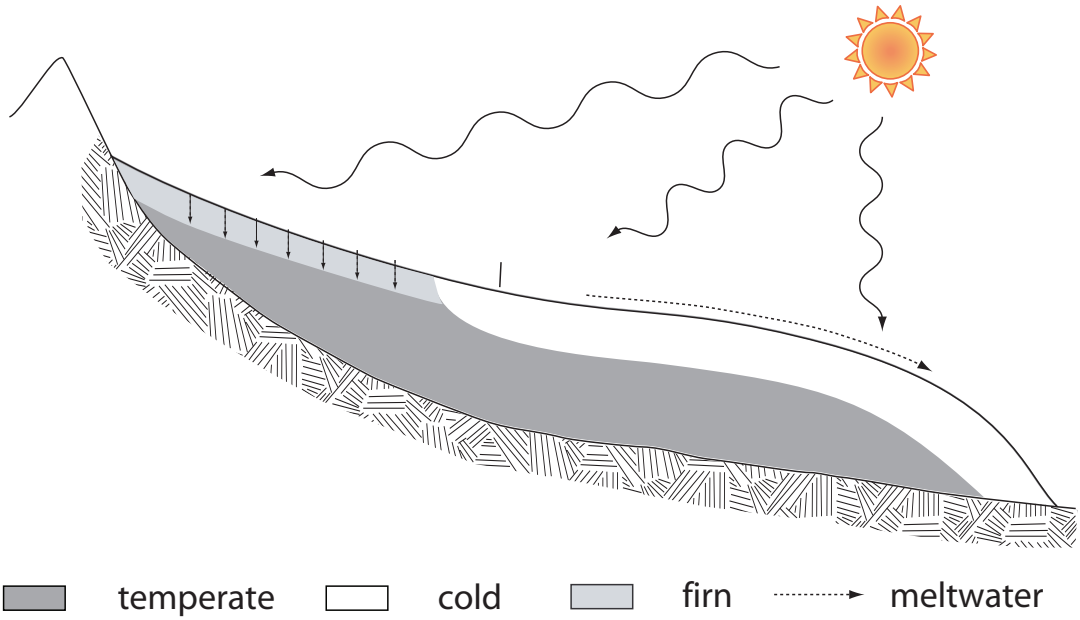


- Scandinavia
- Svalbard
- Rocky Mountains
- Alaska
- Antarctic Peninsula

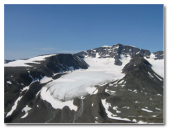
Scandinavian-type thermal structure

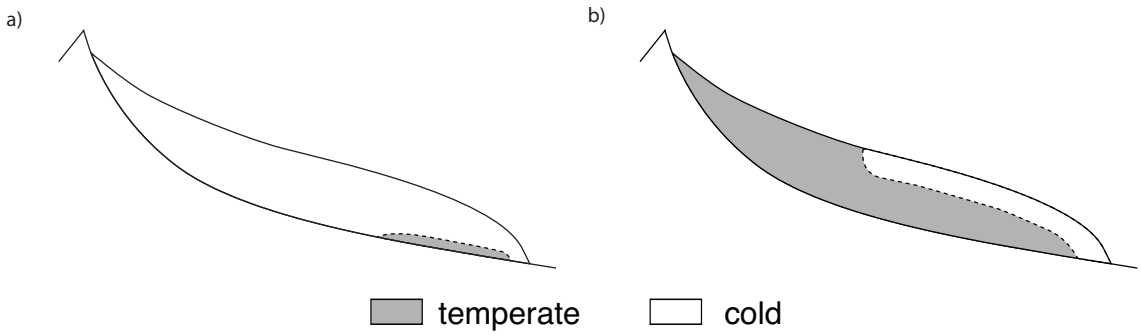
Why is the surface layer in the ablation area cold? Isn't this counter-intuitive?





Canadian-type thermal structure





- high Arctic latitudes in Canada
- Alaska
- both ice sheets Greenland and Antartica

Thermodyamics in ice sheet models

- only few glaciers are completely cold
- most ice sheet models are so-called *cold-ice method* models
- so far two polythermal ice sheet models

$$\rho c(T)\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = \nabla\cdot k\nabla T + Q$$
$$\rho L\left(\frac{\partial \omega}{\partial t} + \mathbf{v}\cdot\nabla \omega\right) = Q$$

or

$$\rho\left(\frac{\partial E}{\partial t} + \mathbf{v}\cdot\nabla E\right) = \nabla\cdot \nu\nabla E + Q$$

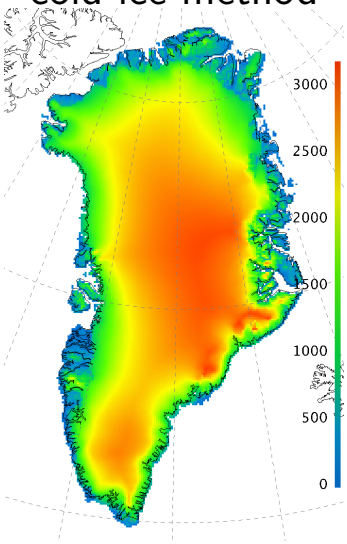
Thermodyamics in ice sheet models

Cold vs Polythermal for Greenland

- thinner temperate layer
- but difference in total ice volume for steady-state run is $< 1\%$ (Greve, 1995)
- SO WHAT?
- better conservation of energy
- temperate basal ice means ice is sliding at the base
- new areas may become temperate

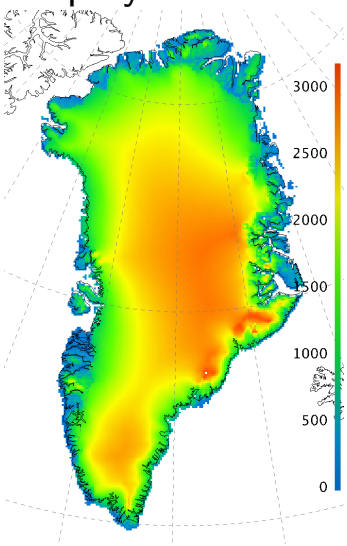
Thermodyamics in ice sheet models

cold-ice method



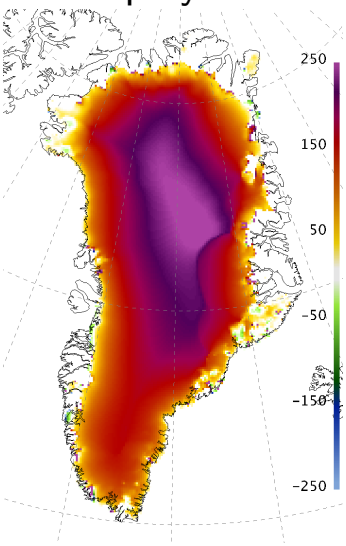
3.295×10^6kg

polythermal



3.025×10^6kg

cold - polythermal



0.270×10^6kg ($\approx 8\%$)

- ice upper surface elevation (masl) from 10km non-sliding SIA equilibrium run