

Continuum mechanics exercises

Problem 1: Practice with notation

- Write out the expression $v_{i,j}v_j$ that occurs in the momentum balance
- Check expression 2.23 by writing it out in component form

Problem 2: Tensors

Angular momentum conservation implies that the stress tensor is symmetric, and that means that it can be diagonalized. That is, there is an orthogonal transformation (a rotation of the coordinate system) that makes the tensor diagonal. The values in the diagonal are the eigenvalues (principal stresses), and the directions in which they act are given by the eigenvectors. The stress field near the margin of a glacier can be approximated by 2D simple shear:

$$\mathbf{t} = \begin{pmatrix} 0 & \tau \\ \tau & 0 \end{pmatrix} \quad (1)$$

Find the principal stresses and the directions and state what this means for crevasse orientation.

Problem 3: Electrically charged ice?

Assume that ice was electrically charged with a charge density ρ_c . Use the general balance law (eqn. 2.8) to write down an equation that expresses the conservation of electrical charge.

Problem 4: Accelerating ice flow

Estimate the relative size of the acceleration term in the momentum balance and state whether the Stokes approximation appears to be suitable for glaciological purposes.

Problem 5: Flow law

Show that the tensor form of the flow law reduces to (2.33) for simple shear.

Problem 6: Navier-Stokes equation

Use the momentum balance derived in the manuscript and replace the stresses with strain rates to write down the Navier-Stokes equation for slow non-linear fluids.

Reduce this equation to a simpler one for flow through a glacier cross-section. Assume that there is only one non-zero velocity component ($\mathbf{v} = (u, 0, 0)^T$), and all out-of-plane gradients are zero ($\frac{\partial}{\partial x} = 0$).

Problem 7: Energy conservation

At Jakobshavn Isbræ ice is discharged into the ocean with velocities of almost 40 m d^{-1} through a cross section of about 5 km^2 . Let's assume that this ice originates, on average, from about 2500 m a.s.l. Use these numbers to come up with an upper limit for melt water production due to frictional heating. Hint: Don't look at the equations in the manuscript too closely, just think about conservation of energy.