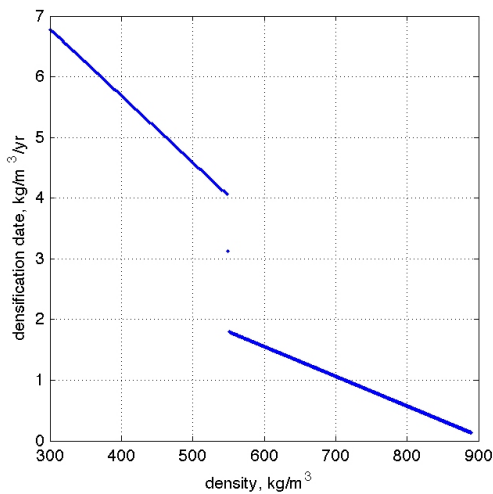
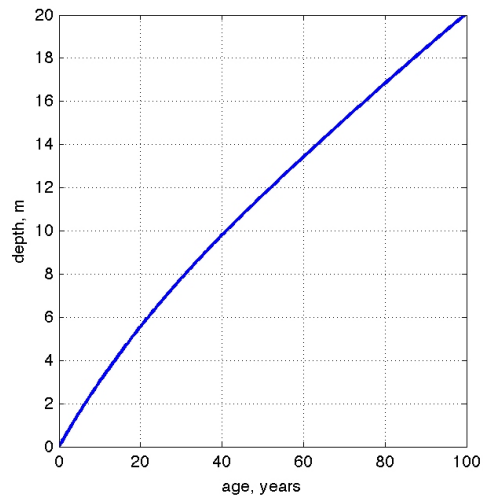
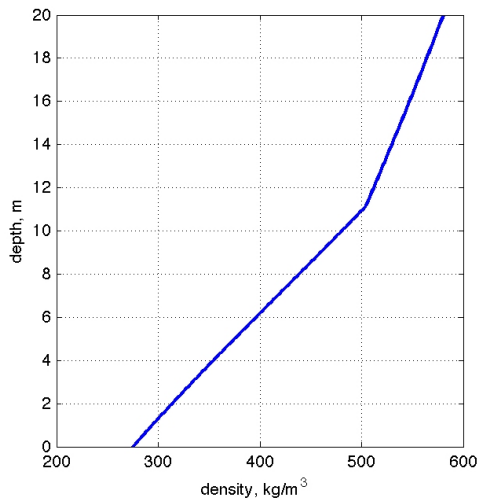


Geodetic estimates of mass balance.

1. Consider a cold firn column, with a density-depth profile shown below. The accumulation rate is 0.1 m/a (ice equivalent), and the surface density is 280 kg m<sup>-3</sup>. For convenience, the steady-state depth-age scale, and densification rate as a function of density are shown as well.

Suppose that a large storm at time zero adds an extra 1 meter (ice equivalent) of snow to the top of the firn column. Thereafter, accumulation proceeds at a steady 0.1 m/a.

- Sketch the density profile immediately after the storm.
- Assume that the densification proceeds as  $d\rho/dt = F(\rho)$ . Write an expression for the rate of change in the thickness of the layer in terms of  $F$  and its derivatives (i) at the surface, and (ii) after it has been buried by 50 years' accumulation.
- Based on (b), sketch the density profile after 50 years. Describe assumptions you have made.
- Under these assumptions, describe what would happen if, instead, there were a ten-year hiatus in accumulation.
- Repeat (briefly) the calculation in b if the year-1 accumulation were equal to 0.1 m/a ice equivalent, but the density was 150 kg m<sup>-3</sup>.



## 2. ICESat elevation-change estimates.

- a. In a region where the accumulation rate is 0.25 m/yr, water equivalent, and the interannual variability in the accumulation rate is 40%, what is the expected year-to-year surface height variability in a steady climate?
- b. What is an order-of-magnitude estimate for the error this variability induces in the cross-track component of ICESat along-track surface-slope estimates? Assume one measurement per year for six years, and 100 m (RMS) offsets between the reference track and ground tracks.
- c. How large are the vertical errors in each measurement due to the slope errors from b)? Compare this variability to that produced by geolocation errors ( $\sim 8$  m) on a 1-degree surface slope.

### 3. Mass-budget estimates

Consider a glacier with an accumulation area that is  $70 \times 300 \text{ km}^2$ , draining through an outlet that is 20 km long and 10 km wide. The ice thickness is known on a 70-km-long cross-glacier profile 10 km upstream of the top of the outlet. The area upstream of the profile is  $20,300 \text{ km}^2$ , the area downstream is  $900 \text{ km}^2$ .

Based on a steady-state climatology of the area, the mean surface mass balance of the area downstream of the profile is estimated at  $-150 \text{ kg m}^{-2} \text{ yr}^{-1}$ .

- a) If the mean surface mass balance upstream of the profile is estimated at  $+200 \text{ kg m}^{-2} \text{ yr}^{-1}$ , and the product of the ice thickness and the surface speed across the profile gives a discharge of 4.5 GT/yr, what is the range of mass-balance estimates for the region upstream of the gate?
- b) Estimate the glacier's net mass balance.
- c) If the mean surface speed at the downstream end of the outlet is measured at  $925 \text{ m yr}^{-1}$ , estimate the ice thickness at the downstream end of the glacier assuming that this part of the glacier is in steady state.
- d) The next year, the surface speed at the downstream end has increased to  $1250 \text{ m yr}^{-1}$ , but the surface mass balance has not changed. Estimate the glacier's net mass balance.
- e) After making these calculations, you learn that the downstream part of the glacier was thinning, both before and during year 1, with a mean rate of  $0.5 \text{ m yr}^{-1}$ , and at a rate near the ice front of  $2 \text{ m yr}^{-1}$ . Assuming that the surface mass balance estimate is correct, suggest a mechanism for this thinning. How does this affect your discharge estimates for years 1 and 2?

#### 4. Ice shelf mass balance

a) Write out the vertical hydrostatic balance for an ice shelf floating in ocean water if mean density of the ice together with the firn layer atop it is  $\rho$ , the density of the sea water is  $\rho_w$ . Rewrite this in terms of the total air content of the firn.

b) Suppose the mean density is  $902 \text{ kg/m}^3$  and the total glacier thickness is  $800 \text{ m}$ . How does the surface height change if a  $1\text{-meter}$  layer of snow of density  $350 \text{ kg/m}^3$  falls on the surface?

c) Suppose the ice speed and surface height are only known  $5 \text{ km}$  downstream of the grounding line, and the melt rate between  $5$  and  $10 \text{ km}$  downstream of the grounding line is estimated at  $4 \text{ m yr}^{-1}$ . If the surface speed is  $1 \text{ km yr}^{-1}$ , what is the minimum contribution of near-grounding-line melt to the ice shelf mass balance?

d) Suppose rifts are observed in the side of an ice shelf that is  $25 \text{ km}$  wide. The rifts are triangular; when first observed,  $60 \text{ km}$  from the ice front, they are  $10 \text{ km}$  long (in the across-flow direction) and  $0.5 \text{ km}$  wide at the widest. The last rift before the ice edge is  $20 \text{ km}$  long and  $2 \text{ km}$  wide. The speed at the front of the ice shelf is  $600 \text{ m yr}^{-1}$ , and there are  $4$  rifts. If the rifts are ignored when the ice shelf mass balance is calculated, what is the resulting error?