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Exercises: Glacial geology

1. Glacier thickness. We wish to estimate the local thickness of a glacier given only a topographic map of the glacier. Assume that the basal shear stress τ_b = 0.8 bars = 8x10^4 Pa, and that the walls are far enough away (the glacier is wide enough) that they play no role in supporting the ice. The contours on the map show that the ice slope is 10m/km or 0.01 at this location. Given the density of ice ρ = 917 kg/m^3 and g = 9.8 m/s^2, estimate the ice thickness H at this location.

The basal shear stress is given by:

$$t = r_i g H \sin q$$

where ρ_i is the ice density, g is the acceleration due to gravity, H is the thickness of the ice, and θ is the ice surface slope angle.
Rearrange:

$$H = \frac{t}{r_i g \sin q}$$

Inserting values:

$$H = \frac{8x10^4\,Pa}{\left(918\,\frac{kg}{m^3}\right)\left(9.8\,\frac{m}{s^2}\right)(0.01)}$$

Note that we are using the approximation that sinθ = tanθ for small angles. Knowing that 1Pa = 1 $\frac{kg\,m}{m^2 s^2}$, we get H = 890 m.

Alternatively, calculate the ice thickness directly from Glen's flow law. You have also measured the ice surface speed to be 80 m/yr. Assume that the ice is not sliding, that all of the ice discharge is accomplished by internal deformation. You will have to assume a flow law parameter, A, and flow law power, n. Take the latter to be n=3, and set A = 6.7x10^{-17} Pa^{-3} yr^{-1}.

From evaluation of the velocity profile fro U(H), as derived from applying Glen's flow law, we obtain

$$U_s = \frac{A}{4}\left(\rho_i g S\right)^3 H^4$$

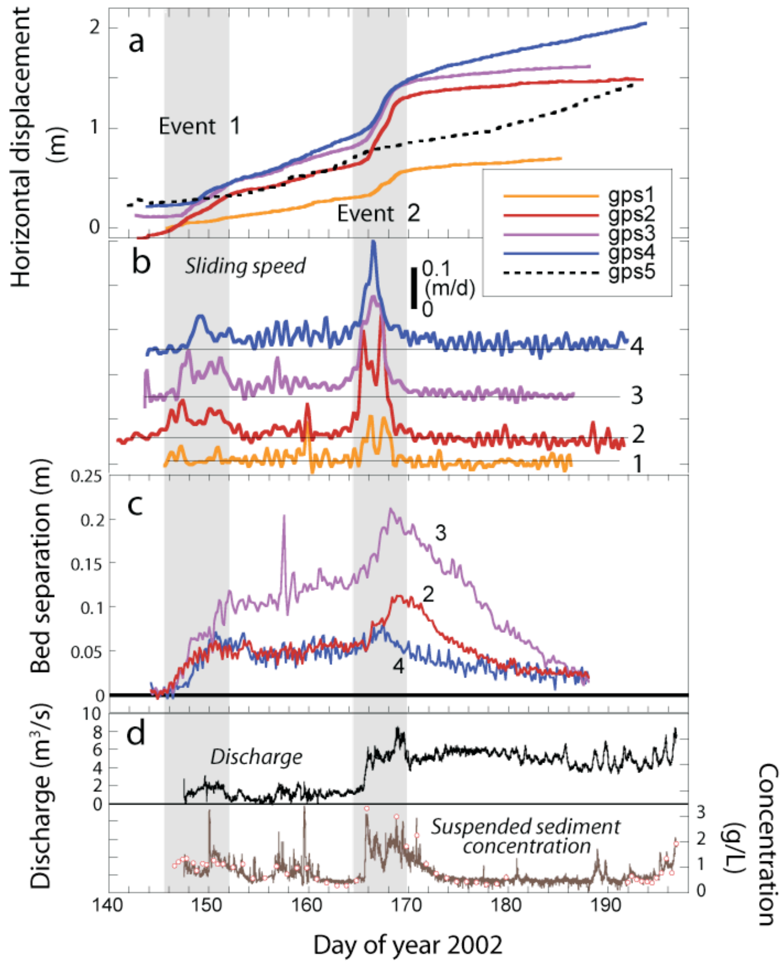
We know U_s, the density, g, and the surface slope, S, and are given A and n. Solve for the one unknown H:

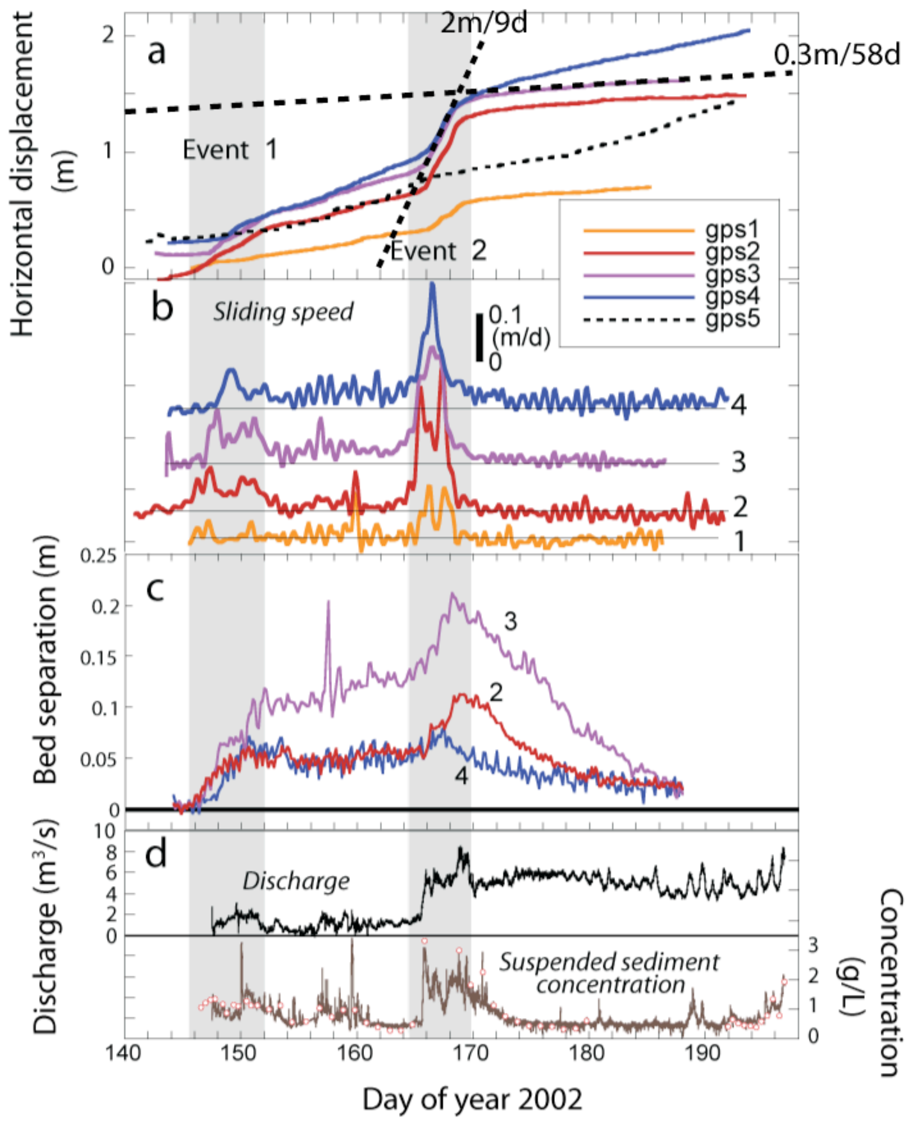
$$H = \left[\frac{4U_s}{A\left(\rho_i g S\right)^3}\right]^{1/4} = \left[\frac{4x80}{6.7x10^{-17}\left(917x9.8x0.01\right)^3}\right]^{1/4} = \left[\frac{3.2x10^2}{6.7x10^{-17}x7.1x10^5}\right]^{1/4}$$

$$= \left[\frac{0.067x10^2}{10^{-17}x10^5}\right]^{1/4} = \left[6.7x10^{12}\right]^{1/4} = 1.6x10^3 m = 1600m$$

Think about why these answers differ – by inspecting the assumptions behind each calculation.

2. From the plot of GPS-derived horizontal speeds on the Bench Glacier below, estimate 1) the background speed of the glacier that is presumably attributable to internal deformation of the ice, and 2) scale the maximum sliding speeds during the two speed-up events at the GPS 3 site.





We have annotated the plot to show two lines with reported slopes: one during sliding event 2, the other with minimal background speed, presumably during motion dominated by internal deformation only. The maximum sliding speed can be deduced from subtracting the internal deformation component from the time of maximum surface speed. Converting the slopes to fractional meters/day, we get: U_{slide} = 0.220 m/d – 0.005 m/d = 0.215 m/d, or about 21 cm/day of sliding during this second fastest event.

And from the decline in bed separation in part C of the plot, determine a characteristic decay timescale that is relevant to cavity collapse after the 2nd sliding event.

The bed separation falls to roughly 1/e of its original maximum separation in about 7-10 days (depending upon which GPS monument you track.

Given this time scale and the thickness of ice measured at GPS3 (=180 m), does this make sense? How might this timescale differ if the ice was say 2x thicker and the same drop in water pressure occurred?

If timescales of collapse scale as stress to the n power, where n is the power in the Glenn flow law, then doubling the thickness should result in $1/(2^3) = 1/8$ for the timescale, or about 1 day.

3. Glacial terminus position. Given a simple glacial valley with a slope of 0.01, such that the elevation of the bed *z = 3500 – 0.01x*, a uniform valley width, and a mass balance profile *b(z)* = 0.01*(*z-ELA*), where the equilibrium line elevation *ELA* = 3000 m, calculate the expected terminus position for a steady glacier. The maximum elevation of the valley, at *x* = 0, is 3500 m. Calculate and plot the down-valley pattern of ice discharge, *Q*, if the glacier is in steady state. (Qualitatively, how would this change if the valley narrowed in a specified pattern down-valley?) If the erosion rate of the glacier is proportional to the ice discharge, sketch how the valley profile will change.

We know that at steady state, in a uniform width glacier, the ice discharge is simply the integral of the mass balance profile.

$$Q = \int b(x)\,dx$$

The mass balance profile *b(z)* that we provided must therefore be converted to a profile in *x*, which is easily done using the two equations shown:

$$b(z) = 0.01(z - ELA)$$

$$ELA = 3000m$$

$$z = 3500 - 0.01x$$

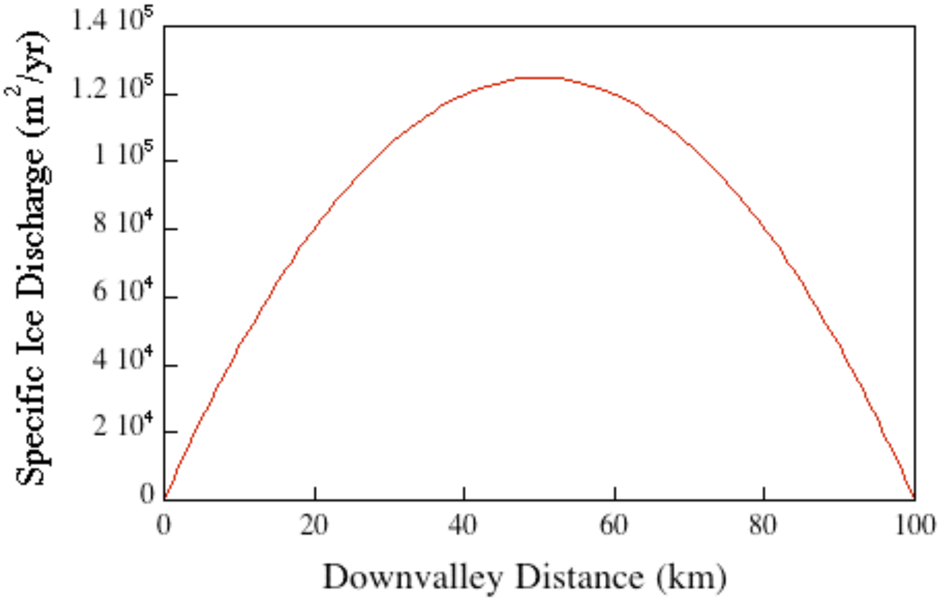
$$b(x) = 0.01((3500 - 0.01x) - ELA) = 0.01((3500 - 0.01x) - 3000)$$

$$b(x) = 0.01(500 - 0.01x) = 5 - 10^{-4}x$$

The integral of this is

$$Q = \int b(x)\,dx = \int (5 - 10^{-4}x)\,dx = 5x + 10^{-4}\frac{x^2}{2}$$

which is an equation for a parabola. This is plotted below:



Note that the terminus, where Q=0, is found at 100 km (=10^5 m). We can find that analytically by setting Q=0 in the equation above:

$$Q = 0 = 5x - 0.5 * 10^{-4} x^2$$

$$5x = 0.5 * 10^{-4} x^2$$

$$5 = 0.5 * 10^{-4} x$$

$$x = 10^5 m = 100 km$$

4. Glacial erosion rates deduced from glacial sediment discharge. Total annual sediment discharge measured from the Bench Glacier by integrating the product of QC through the melt season is 18,200 tonnes (Riihimaki et al., 2005). Given that the Bench Glacier is 7 km long and 1 km wide, calculate the mean annual subglacial erosion rate. Given that the annual sliding distance is 1.5 m, calculate how efficient the erosional engine is (=erosion rate/sliding rate).

Ref: Riihimaki, C. A., K. R. MacGregor, R. S. Anderson, S. P. Anderson, and M. G. Loso (2005), Sediment evacuation and glacial erosion rates at a small alpine glacier, J. Geophys. Res., 110, F03003, doi:10.1029/2004JF000189.

Erosion rate is 1.04 mm/yr is we assume that the bedrock density is 2500 kg/m^3.

Efficiency = Erosion rate/sliding speed = 1.04 mm/yr / 1500 mm/yr = 7×10^{-4}

5. Hyperpycnal flows and the fate of glacial sediment in seawater. Calculate the concentration of quartz sediment necessary to make the bulk density of river water equal to that of seawater (1003 kg/m^3). Concentrations exceeding this will generate hyperpycnal flows.
On Bench Glacier's river, we measured concentrations as high as 3g/L. Would these be hyperpycnal upon entering the sea?

Here the problem is calculating the density of a mixture of two substances. Call the concentration of quartz C_q. Then the density of a mixture of fresh water and quartz becomes:

$$\rho = (\rho_w(1 - C_q) + \rho_q(C_q)) = \rho_w + C_q(\rho_q - \rho_w)$$

We wish to solve this for C_q required to obtain ρ=1003 kg/m^3.

$$\rho = (\rho_w(1 - C_q) + \rho_q(C_q)) = \rho_w + C_q(\rho_q - \rho_w)$$

$$C_q = \frac{\rho - \rho_w}{\rho_q - \rho_w} = \frac{1003 - 1000}{2650 - 1000} = 0.0018 = 0.18\%$$