# Exercise

#### 1 Flow speeds

Will and Carl are two grad students in glaciology who are really excited about their first opportunity to do field work on Kennicott Glacier, Alaska. Arriving in the town of McCarthy, their spirits are slightly dampened by heavy fog that makes it impossible to see Root Glacier from their tent. They decide to get started by going through the scribbles left by former grad student Tim. In Tim's field book they discover a sketch of Kennicott and Root Glacier (see Figure 1) and a note that the average ice thickness is 900 m. Later that evening, at the bar, Will and Carl make a bet who can better guess the surface speed of Kennicott Glacier between Hidden Creek Lake and Donahue Falls Lake. Recalling the glacier dynamics course they took the previous semester, Will assumes that Root Glacier can be approximated by an inclined parallel-sided slab and that half of the surface motion is due to basal sliding. Carl, on the other hand, opts for a cylindrical channel.

**Question** During the second day, Will and Carl find a hard disk with GPS velocities from GPS station 3. They're surprised to discover that the average surface speed is around 0.5 meters per day. Who made the better guess and won the beer at the bar that night?

Hint: assume a rate factor  $A = 2.4 \cdot 10^{-24} \,\mathrm{s}^{-1} \,\mathrm{Pa}^{-3}$ 

#### 2 Mass flux

Fortunately the weather clears up and Will and Carl spend the better part of a week collecting additional ice thickness measurments with their ground penetrating radar. Since it's late summer, they have to carry the radar around by hand, a really laborious task. They conclude that collecting radar data is easier in spring when you can ski or snowmachine, simply draggin the radar behind. Nonetheless, with an ice thickness profile



Figure 1: Map from *Bartholomaus et al.* (2008)

across the glacier, passing through GPS site 3 and the velocity readings from the same GPS, Will and Carl intend to calculate the mass flux through site 3. They know that the mass flux Q is given by

$$Q = \bar{v}H = \int_{0}^{H} v \,\mathrm{d}z,\tag{1}$$

where  $\bar{v}$  is the vertically-averaged horizontal flow speed and H is the ice thickness. Will and Carl only know the surface speed but not the depth distribution. Will realizes that in the case of plug flow (i.e. all horizontal motion is due to basal sliding) the verticallyaveraged velocity is equal to the surface velocity. Carl then suggests that in the case of no basal sliding (i.e. vertical shearing only), the vertically-averaged horizontal velocity should be lower than for plug flow.

**Question** Is Carl right, and if so, by how much?

## 3 Surface evolution

Assume a cold glacier (no basal melt), and that the base of the glacier is not moving (i.e.  $\partial b/\partial t = 0$ ). The evolution of the ice surface, h, is then given by

$$\frac{\partial h}{\partial t} = -\nabla \cdot \mathbf{Q} + a_{\rm s},\tag{2}$$

where  $\nabla \cdot \mathbf{Q}$  and  $a_s$  are the horizontal flux divergence and the climatic mass balance (i.e. sum of surface and internal mass balances), respectively.

- 1. Explain the meaning of the above terms.
- 2. If the flux divergence is constant throughout the year, describe the evolution of the ice surface over a hydrologic year (September to September):
  - in the accumulation zone. Use the following values:  $\nabla \cdot \mathbf{Q} = 2 \operatorname{m} \operatorname{a}^{-1}$ ,  $a_{\mathrm{s}}^{\mathrm{summer}} = 4 \operatorname{m} \operatorname{a}^{-1}$ ,  $a_{\mathrm{s}}^{\mathrm{summer}} = -1 \operatorname{m} \operatorname{a}^{-1}$ .
  - in the ablation zone. Use the following values:  $\nabla \cdot \mathbf{Q} = -1 \,\mathrm{m \, a^{-1}}, a_{\mathrm{s}}^{\mathrm{winter}} = 1 \,\mathrm{m \, a^{-1}}, a_{\mathrm{s}}^{\mathrm{summer}} = -3 \,\mathrm{m \, a^{-1}}.$
  - What happens at the equilibrium line?
- 3. The 1912 eruption of Katmai volcano, Alaska, covered a nearby glacier completely with debris, thereby effectively shielding the glacier from ablation and accumulation. What will happen to this glacier?

#### 4 Mass balance, surface evolution, and vertical velocity

Will and Carl come across a paper by *Konrad et al.* (1999) that says: "The spatial mass balance is the change of elevation with time (i.e., vertical velocity) at every point on the surface." Think about it.

**Question** What are the authors trying to say? Can you help them getting it right? What does repeated laser altimetry measure, mass balance, surface elevation, or vertical velocity?

## 5 Climate history

Air temperatures in Alaska was oscillating with a period of about 50 years and an amplitude of about 2 °C between 1950 and 2000.

**Question** How deep down would you be able to detect such temperature variation in stagnant ice if the accuracy of your temperature sensors are 0.01 K assuming an ice temperature of  $-3^{\circ}$  C?

## 6 Cold content

Carl decided that snow is much more interesting the glacier ice. On a sunny morning in late March somewhere outside of Fairbanks, he measures the temperature profile in the snow pack after a cold night (Figure 2). During the day air temperatures increase above the freezing point and it starts raining. The cold content can be eliminated by release of energy from refreezing water.

**Question** How much melt in mm w.e. (water equivalent) or  $kg/m^2$  is needed to completely eliminate the cold content? Hint: Make a reasonable guess for an anverage snow density.

## 7 Melting temperature depression

What is the pressure melting temperature at the base of Gornergletscher (Figure 3)? What does the Clausius-Clapeyron relation indicate in terms of air-saturation of the meltwater? The pressure p is the sum of the hydrostatic pressure and the atmospheric pressure,  $p = \rho g H + p_{\text{atm}}$ . Assume  $p_{\text{atm}} = 75 \text{ kPa}$ .



Figure 2: Snow temperature profile



**Figure 3:** Cooling of a borehole drilled in the confluence area of Gorner-/Grenzgletscher. Temperatures measured every day after completion of drilling are shown in increasingly lighter colors, and after three months (leftmost yellow curve). Data from *Ryser* (2009).

## 8 Lake Vostok

- 1. Describe 2 different ways how heat can be moved through a polar ice sheet.
- 2. What is the Péclet Number, and how is it useful?
- 3. The coldest temperature ever recorded is -89° C at Vostok in East Antarctica (in July 1983). The mean annual temperature is -55° C. However, deep under the ice is lake Vostok, a lake of the size of lake Ontario. Calculate the minimum geothermal flux needed for a lake to form. Possibly relevant quantities:
  - Surface elevation 3488 m
  - $\bullet\,$  Ice thickness  $3300\,\mathrm{m}$
  - Snow accumulation rate  $2 \,\mathrm{cm} \,\mathrm{a}^{-1}$  (water equivalent)
  - A reasonable average thermal conductivity for the cold temperatures of the East Antarctic Ice Sheet is  $k = 2.5 \,\mathrm{W}\,\mathrm{m}^1\,\mathrm{K}^1$ .

#### References

- Bartholomaus, T. C., R. S. Anderson, and S. P. Anderson (2008), Response of glacier basal motion to transient water storage, *Nat. Geosci.*, 1(1), 33–37, doi: 10.1038/ngeo.2007.52.
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